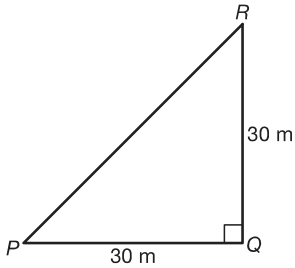
06 Hoeken en afstanden

6.1 Kijkhoeken

bladzijde 70

1 **a**



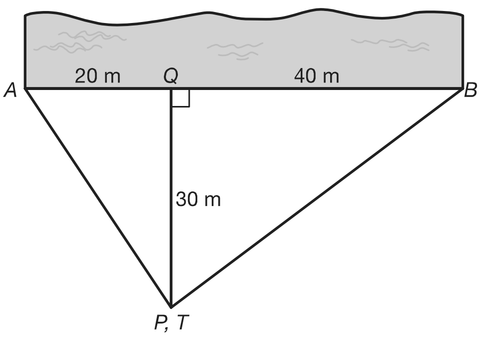
ZORG ER VOOR DAT JE GRAFISCHE REKENMACHINE OP GRADEN IS INGESTELD.

In $\triangle PQR$ is $\angle P = 45°$. Houd je rekening met Paula's ooghoogte dan zal de hoek waaronder Paula QR ziet iets groter dan 45° zijn.

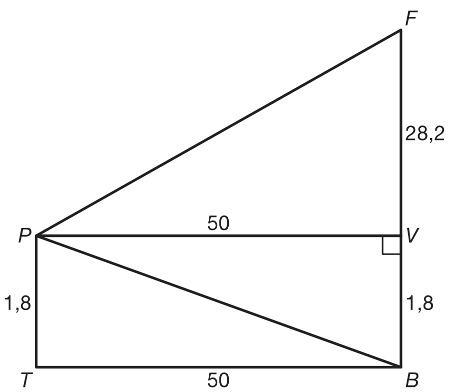
b De randen AE en BF zijn in werkelijkheid even groot. Omdat Paula dichter bij AE staat vergeleken met BF ziet ze AE onder een grotere hoek vergeleken met BF.
Dus ziet Paula BF onder de kleinste hoek.

bladzijde 71

2



De stelling van Pythagoras in $\triangle QTB$: $BT = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50$ m.



In $\triangle BPV$ is $\tan \angle BPV = \dfrac{1{,}8}{50}$, dus $\angle BPV \approx 2{,}06°$

In $\triangle FPV$ is $\tan \angle FPV = \dfrac{28{,}2}{50}$, dus $\angle FPV \approx 29{,}42°$

$\angle BPF = \angle BPV + \angle FPV \approx 2{,}06° + 29{,}42 \approx 31{,}48°$

De gevraagde kijkhoek is 31,5°.

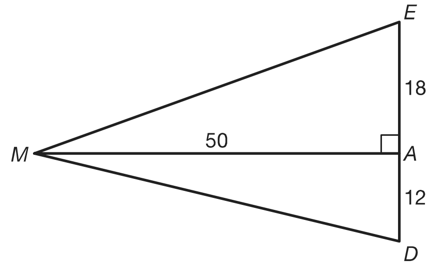
3 **a** $\tan \angle EMA = \frac{18}{50}$

$\angle EMA \approx 19{,}80°$

$\tan \angle DMA = \frac{12}{50}$

$\angle DMA \approx 13{,}50°$

De kijkhoek $\angle EMD \approx 33{,}3°$



b In bovenaanzicht geeft de stelling van Pythagoras

$MC^2 = AM^2 + AC^2$

$MC^2 = 2500 + 1600 = 4100$

$\text{MC} = \sqrt{4100} \approx 64{,}03\,\text{m}$

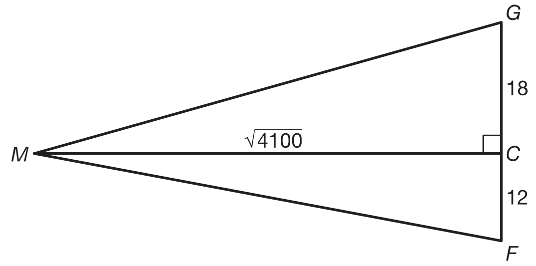
$\tan \angle \text{GMC} = \dfrac{18}{\sqrt{4100}}$

$\angle GMC \approx 15{,}70°$

$\tan \angle \text{FMC} = \dfrac{12}{\sqrt{4100}}$

$\angle FMC \approx 10{,}61°$

De kijkhoek $\angle GMF \approx 26{,}3°$



4 **a** Trek $CF \perp BE$.

$BF = 1{,}8$, $DF = 2{,}2$, $CF = 20$

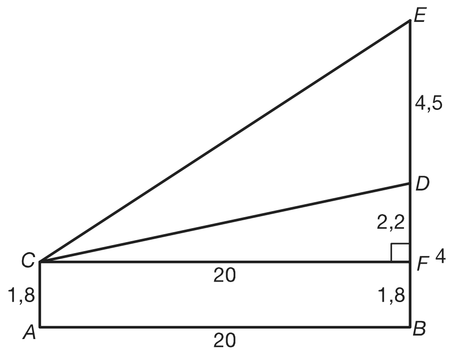
In $\triangle CFD$: $\tan \angle DCF = \dfrac{2{,}2}{20}$

$\angle DCF \approx 6{,}28°$

In $\triangle CFE$: $\tan \angle ECF = \dfrac{6{,}7}{20}$

$\angle ECF \approx 18{,}52°$

$\beta = \angle ECF - \angle DCF \approx 12{,}2°$



b Nu $AB = CF = 10$. Dit geeft

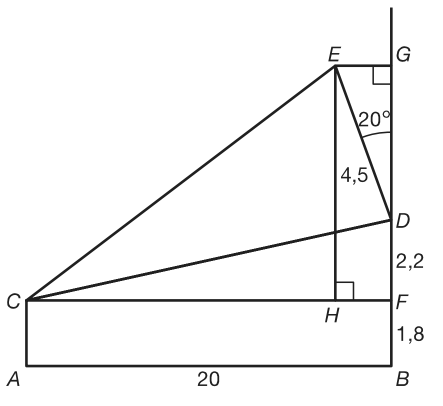
$\tan \angle DCF = \dfrac{2{,}2}{10}$, dus $\angle DCF \approx 12{,}41°$

$\tan \angle ECF = \dfrac{6{,}7}{10}$, dus $\angle ECF \approx 33{,}82°$.

Dus $\beta \approx 21{,}4°$. Dit is niet $2 \times 12{,}2 \approx 24{,}5$.

Karel heeft dus geen gelijk.

c



Trek $EG \perp BD$ en $EH \perp CF$.

In $\triangle DEG$ is $\sin 20° = \dfrac{EG}{4{,}5}$, dus $EG = 4{,}5 \sin 20° \approx 1{,}54$.

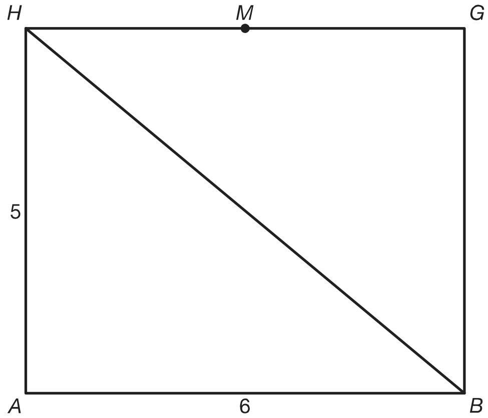
Dit geeft $CH \approx 20 - 1{,}54 \approx 18{,}46$.

$\cos 20° = \frac{DG}{4{,}5}$, dus $DG = 4{,}5 \cos 20° \approx 4{,}23$. Dit geeft $EH = FG \approx 6{,}43$.
In $\triangle CHE$ is $\tan \angle ECH = \frac{EH}{CH} = \frac{6{,}43}{18{,}46} \approx 0{,}35$, dus $\angle ECH \approx 19{,}20°$.
Dit geeft $\beta = \angle ECH - \angle DCF \approx 19{,}20° - 6{,}28° \approx 13°$.

bladzijde 72

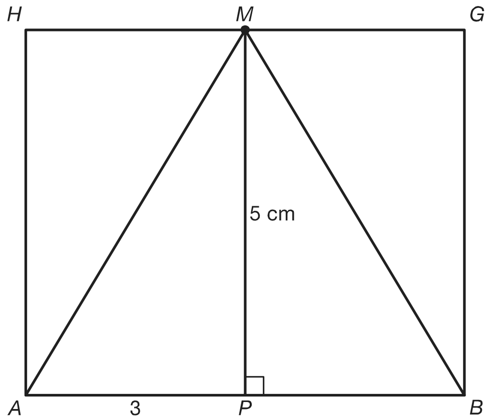
5 **a** Het vlak *TQR*.
b Het vlak *TBF*.

6 **a** In $\triangle ADH$ geeft de stelling van Pythagoras $AH = 5$



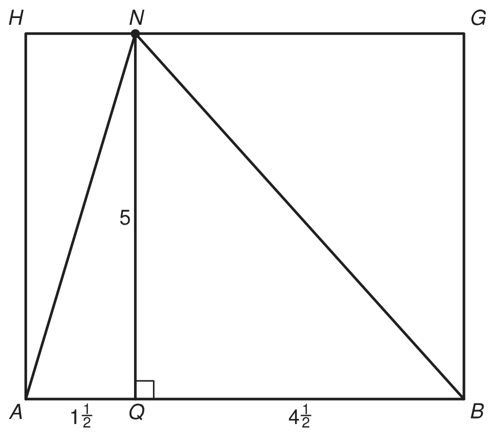
$\tan \angle AHB = \frac{6}{5}$
$\angle AHB \approx 50°$

b



Trek $MP \perp AB$.
$\tan\left(\frac{1}{2}\angle AMB\right) = \frac{3}{5}$
$\frac{1}{2}\angle AMB \approx 30{,}96°$
$\angle AMB \approx 62°$

c



Trek $NQ \perp AB$.
$\tan \angle ANQ = \frac{1\frac{1}{2}}{5}$

$\angle ANQ \approx 16{,}70°$

$\tan \angle BNQ = \frac{4\frac{1}{2}}{5}$

$\angle BNQ \approx 41{,}99°$

$\angle ANB = \angle ANQ + \angle BNQ \approx 58{,}69°$

Het gemiddelde van $\angle AMB$ en $\angle AHB$ is ongeveer 56°.

Daan heeft dus geen gelijk.

bladzijde 73

7 **a** In grondvlak: $AC = \sqrt{32}$ en $AS = \frac{1}{2}\sqrt{32}$

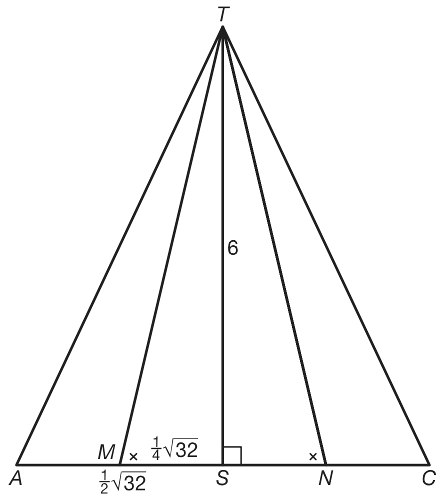
$\tan \angle CAT = \frac{6}{\frac{1}{2}\sqrt{32}}$

$\angle CAT \approx 65°$

$\tan \angle CMT = \frac{6}{\frac{1}{4}\sqrt{32}}$

$\angle CMT \approx 77°$

$\angle CNT = 180° - \angle ANT$
$= 180° - \angle CMT$
$\approx 180° - 77° = 103°$



b In $\triangle PST$: $\tan 70° = \frac{6}{PS}$

$PS = \frac{6}{\tan 70°} \approx 2{,}18$

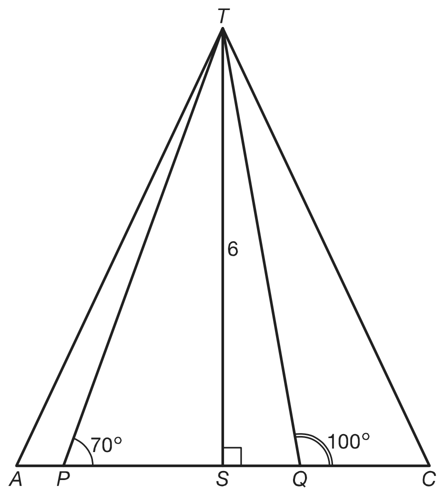
$AP \approx \frac{1}{2}\sqrt{32} - 2{,}18 \approx 0{,}64$

c In $\triangle SQT$ is $\angle SQT = 180° - 100° = 80°$

$\tan 80° = \frac{6}{SQ}$

$SQ = \frac{6}{\tan 80°} \approx 1{,}06$

$AQ \approx \frac{1}{2}\sqrt{32} + 1{,}06 \approx 3{,}89$

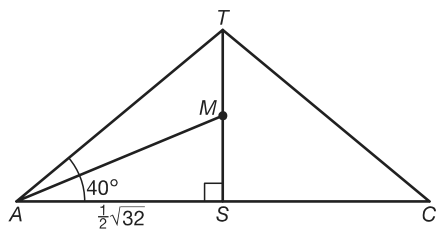


8 **a** In grondvlak: $AC = \sqrt{32}$ en $AS = \frac{1}{2}\sqrt{32}$

In $\triangle AST$: $\cos 40° = \frac{\frac{1}{2}\sqrt{32}}{AT}$

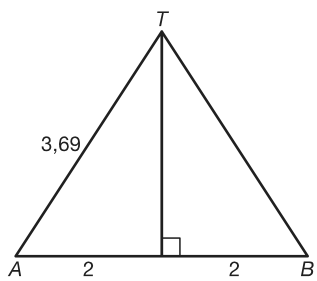
$AT = \frac{\frac{1}{2}\sqrt{32}}{\cos 40°} \approx 3{,}69$

De opstaande ribben zijn 3,69 lang.



b $\cos \angle BAT = \frac{2}{3{,}69}$

$\angle BAT \approx 57°$



c Zie de figuur van a

$\tan 40° = \frac{TS}{\frac{1}{2}\sqrt{32}}$

$TS = \frac{1}{2}\sqrt{32} \cdot \tan 40° \approx 2{,}37$

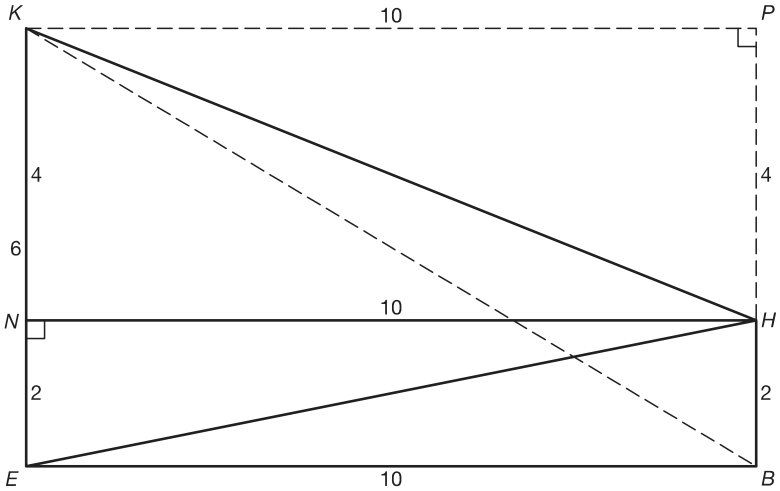
$MS = \frac{1}{2} \cdot TS \approx 1{,}19$

$\tan \angle AMS \approx \frac{\frac{1}{2}\sqrt{32}}{1{,}19} \approx 2{,}38$

$\angle AMS \approx 67{,}2°$
$\angle AMC = 2 \cdot \angle AMS \approx 134°$

9 **a** $KL \,//\, IH$ en $KL = IH$
Van K naar L gaat 2 naar beneden
dus ook van I naar H 2 naar beneden.
I op hoogte 4, dus H op hoogte 2.
Ook G ligt op hoogte 2.
Dus $AG = BH = 2$.

b $ABCDEF$ is regelmatige zeshoek met zijde 5, dus $BE = 10$.



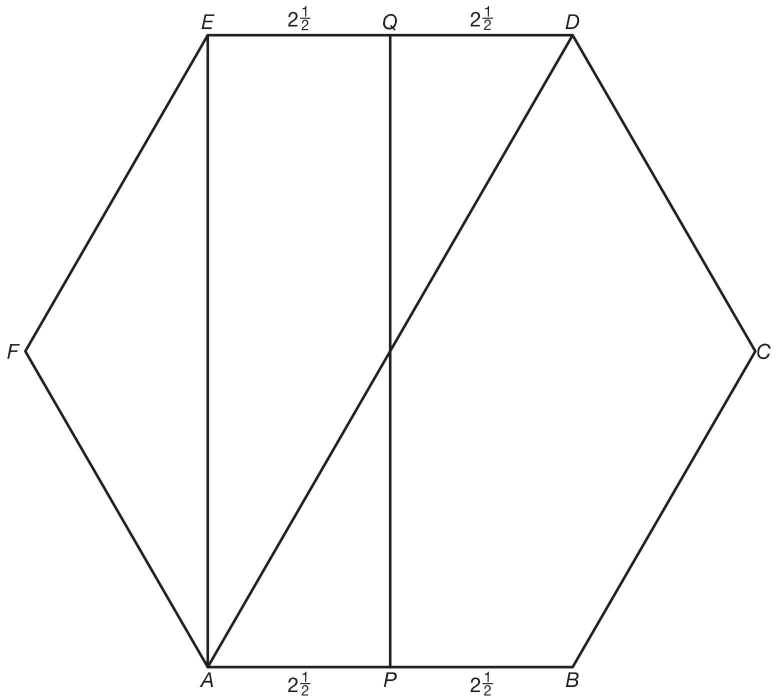
$\tan \angle EHN = \frac{2}{10}$, dus $\angle EHN \approx 11{,}3°$

$\tan \angle KHN = \frac{4}{10}$, dus $\angle KHN \approx 21{,}8°$

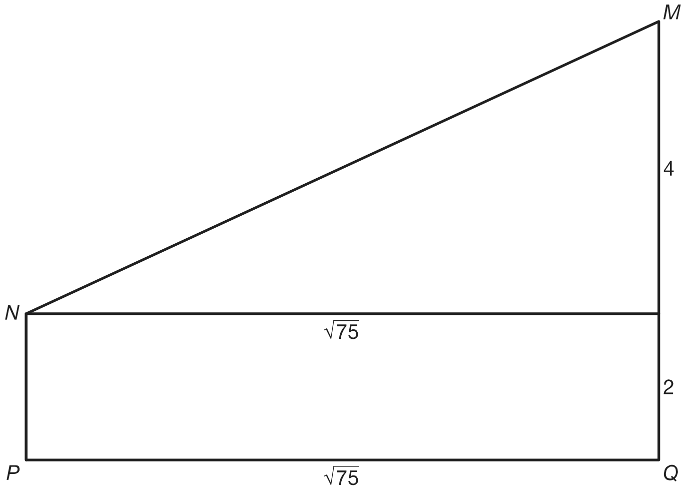
De kijkhoek is dus $11{,}3° + 21{,}8° \approx 33°$.

c $\angle HKP = \angle KHN$ (Z-hoeken) dus ook $\angle HKP \approx 21{,}8°$
$\tan \angle BKP = \frac{6}{10}$, dus $\angle BKP \approx 31{,}0°$
De kijkhoek is dus $31{,}0° - 21{,}8° \approx 9°$.

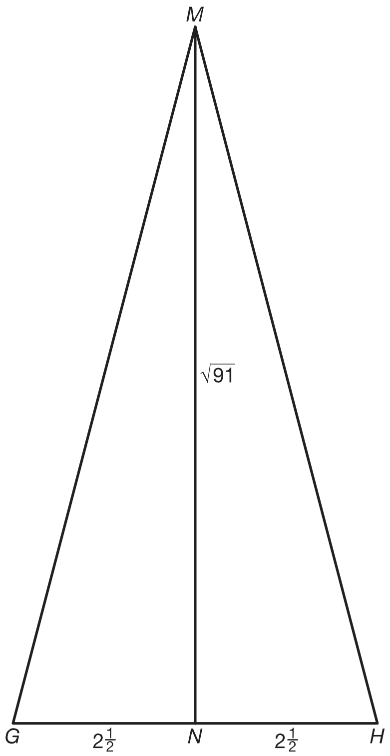
d Het punt N is het midden van GH.
In het grondvlak ligt P recht onder N en Q recht onder M.



De stelling van Pythagoras in $\triangle$ADE: AE $= \sqrt{10^2 - 5^2} = \sqrt{75}$, dus ook PQ $= \sqrt{75}$.



Vlak door *P*, *Q* en *M*
Met de stelling van Pythagoras: MN $= \sqrt{75 + 4^2} = \sqrt{91}$.



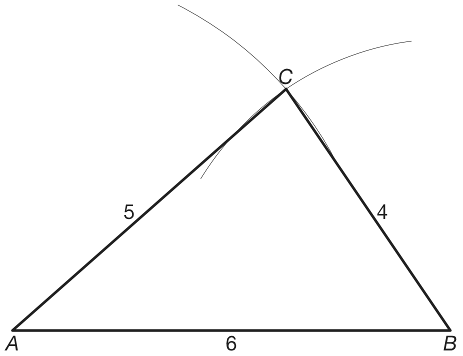
In $\triangle GMN$ geldt: tan $\angle$GMN $= \dfrac{2\frac{1}{2}}{\sqrt{91}}$, dus $\angle GMN \approx 14{,}69°$.

$\angle GMN = 2 \cdot \angle GMN \approx 29°$.

6.2 De cosinusregel

bladzijde 75

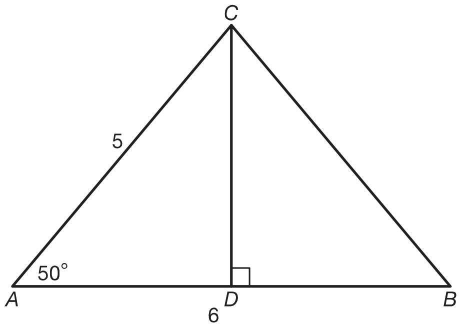
10 **a**



b $\angle A \approx 41{,}4°$, je kunt dus vinden $\angle A \approx 41°$ of $\angle A \approx 42°$.

c Om $\angle A$ te berekenen heb je een rechthoekige driehoek nodig.
De hoogtelijn CD loodrecht op AB ligt voor de hand.
In $\triangle ADC$ zijn de zijden AD en CD onbekend, dus kun je op deze manier $\angle A$ niet berekenen.

11 **a**



b $BC \approx 4{,}74$, je kunt dus vinden $BC \approx 4{,}7$ of $BC \approx 4{,}8$.

c In $\triangle ADC$: $\sin 50° = \dfrac{CD}{5}$, dus $CD = 5 \cdot \sin 50° \approx 3{,}83$

en $\cos 50° = \dfrac{AD}{5}$, dus $AD = 5 \cdot \cos 50° \approx 3{,}21$

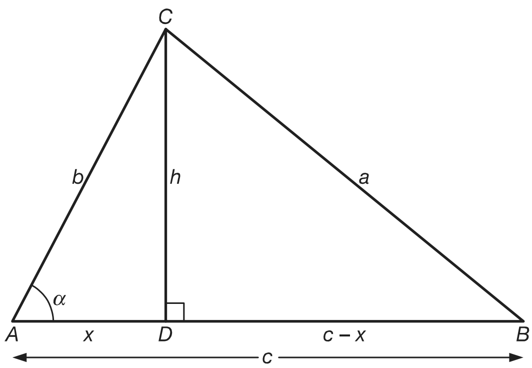
$AB = 6$, dus $BD = 6 - AD \approx 2{,}79$

In $\triangle BDC$ geeft de stelling van Pythagoras $BC^2 = BD^2 + CD^2$,

dus $BC \approx \sqrt{2{,}79^2 + 3{,}83^2} \approx 4{,}74$.

bladzijde 76

12 Gegeven: ΔABC met de hoogtelijn CD.



Te bewijzen: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Bewijs:

De stelling van Pythagoras in $\triangle ADC$ geeft $x^2 + h^2 = b^2$
De stelling van Pythagoras in $\triangle BDC$ geeft $a^2 = (c - x)^2 + h^2 = c^2 - 2cx + x^2 + h^2$ $\Big\}$

Dus $a^2 = b^2 + c^2 - 2cx$
In $\triangle ADC$ is $\cos \alpha = \dfrac{x}{b}$, dus $x = b \cdot \cos \alpha$ $\Big\}$ $a^2 = b^2 + c^2 - 2bc \cos \alpha$

13 $a^2 = b^2 + c^2 - 2bc \cos \alpha$
$16 = 25 + 36 - 2 \cdot 5 \cdot 6 \cos \alpha$
$16 = 61 - 60 \cos \alpha$
$60 \cos \alpha = 45$
$\cos \alpha = \frac{45}{60}$
$\alpha \approx 41{,}4°$
$b^2 = a^2 + c^2 - 2ac \cos \beta$
$25 = 16 + 36 - 2 \cdot 4 \cdot 6 \cos \beta$
$25 = 52 - 48 \cos \beta$
$48 \cos \beta = 27$
$\cos \beta = \frac{27}{48}$
$\beta \approx 55{,}8°$
$\alpha + \beta + \gamma = 180°$ geeft $\gamma = 180° - \alpha - \beta \approx 82{,}8°$
Opmerking: je kunt γ ook met de cosinusregel berekenen.

14 **a** 1 $\cos 10° \approx 0{,}9848$ $\cos 170° \approx -0{,}9848$
2 $\cos 50° \approx 0{,}6428$ $\cos 130° \approx -0{,}6428$
3 $\cos 29{,}3° \approx 0{,}8721$ $\cos 150{,}7° \approx -0{,}8721$
Als $\alpha + \beta = 180°$, dan is $\cos \alpha = -\cos \beta$.

b Noem $CD = h$ en $AD = x$.
In $\triangle ACD$ is $x^2 + h^2 = b^2$
In $\triangle BCD$ is $(c + x)^2 + h^2 = a^2$
$c^2 + 2cx + x^2 + h^2 = a^2$
vervangen van $x^2 + h^2$ door b^2 geeft
$c^2 + 2cx + b^2 = a^2$
dus $a^2 = c^2 + 2cx + b^2$.

In $\triangle ACD$ is $\cos(180° - \alpha) = \frac{x}{b}$, dus $x = b \cos(180° - \alpha) = b \cdot -\cos \alpha = -b \cos \alpha$

Dit invullen in $a^2 = c^2 + 2cx + b^2$ geeft
$a^2 = c^2 + 2c \cdot -b \cos \alpha + b^2$
dus $a^2 = b^2 + c^2 - 2bc \cos \alpha$.

15 $a^2 = b^2 + c^2 - 2bc \cos \alpha$
$16 = 49 + 25 - 2 \cdot 7 \cdot 5 \cdot \cos \alpha$
$16 = 74 - 70 \cos \alpha$
$70 \cos \alpha = 58$
$\cos \alpha = \frac{58}{70}$
$\alpha \approx 34°$
$b^2 = a^2 + c^2 - 2ac \cos \beta$
$49 = 16 + 25 - 2 \cdot 4 \cdot 5 \cdot \cos \beta$
$49 = 41 - 40 \cos \beta$
$40 \cos \beta = -8$
$\cos \beta = \frac{-8}{40}$
$\beta \approx 102°$
$\alpha + \beta + \gamma = 180°$
$\gamma = 180° - \alpha - \beta \approx 44°$
Opmerking: je had γ ook met de cosinusregel kunnen berekenen.

16 **a** $a^2 = b^2 + c^2 - 2bc \cos \angle A$
$64 = 49 + 100 - 2 \cdot 7 \cdot 10 \cdot \cos \angle A$
$64 = 149 - 140 \cos \angle A$
$140 \cos \angle A = 85$
$\cos \angle A = \frac{85}{140}$
$\angle A \approx 52{,}6°$

b In $\triangle ACD$ is $\sin \angle A = \frac{CD}{7}$
$CD = 7 \cdot \sin 52{,}6° \approx 5{,}6$

17 **a** In grondvlak: $AC^2 = 6^2 + 4^2 = 52$, dus $AC = \sqrt{52}$.
In zijvlak $ADHE$: $AH^2 = 4^2 + 3^2 = 25$ dus $AH = 5$.
In achtervlak: $CH^2 = 6^2 + 3^2 = 45$, dus $CH = \sqrt{45}$.

b in $\triangle ACH$: $CH^2 = AC^2 + AH^2 - 2 \cdot AC \cdot AH \cdot \cos \angle A$

$$45 = 52 + 25 - 2 \cdot \sqrt{52} \cdot 5 \cdot \cos \angle A$$
$$45 = 77 - 10\sqrt{52} \cdot \cos \angle A$$
$$10\sqrt{52} \cdot \cos \angle A = 32$$
$$\cos \angle A = \frac{32}{10\sqrt{52}}$$
$$\angle A \approx 64°$$

c In diagonaalvlak $DBFH$ (rechthoek):

$$\tan \angle DBH = \frac{DH}{BD} = \frac{3}{\sqrt{52}}$$
$$\angle DBH \approx 23°$$

De cosinusregel is niet nodig omdat $\triangle DBH$ rechthoekig is.

bladzijde 77

18 **a** In zijvlak $BCGF$: $BG^2 = 6^2 + 6^2 = 72$, dus $BG = \sqrt{72}$.

In bovenvlak: $GM^2 = 6^2 + 3^2 = 45$, dus $GM = \sqrt{45}$.

In het verticale vlak door BM : $BM^2 = \left(\sqrt{45}\right)^2 + 6^2 = 81$, dus $BM = 9$.
In $\triangle BGM$: $BG^2 = GM^2 + BM^2 - 2 \cdot GM \cdot BM \cdot \cos \angle M$

$$72 = 45 + 81 - 2 \cdot \sqrt{45} \cdot 9 \cdot \cos \angle M$$
$$72 = 126 - 18\sqrt{45} \cdot \cos \angle M$$
$$18\sqrt{45} \cdot \cos \angle M = 54$$
$$\cos \angle M = \frac{54}{18\sqrt{45}}$$
$$\angle BMG \approx 63{,}435° \approx 63°$$

b $O(\triangle BGM) = \frac{1}{2} \cdot MB \cdot MG \cdot \sin \angle BMG$

$$= \tfrac{1}{2} \cdot 9 \cdot \sqrt{45} \cdot \sin 63{,}435° \approx 27{,}0$$

19 $PR^2 = PQ^2 + QR^2 - 2 \cdot PQ \cdot QR \cdot \cos \angle PQR$
$= 100 + 49 - 2 \cdot 10 \cdot 7 \cdot \cos 110°$
$\approx 196{,}88$
$PR \approx 14{,}03$

20 **a** In grondvlak (vierkant $ABCD$) geldt $AC^2 = AB^2 + BC^2$
$AC^2 = 25 + 25 = 50$

dus $AC = \sqrt{50}$ en $AS = \frac{1}{2}\sqrt{50}$.

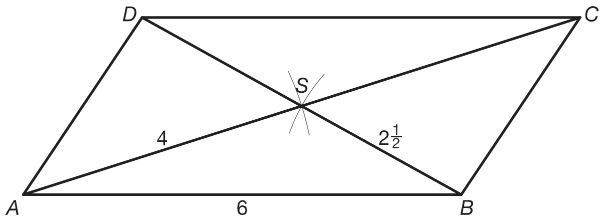
In $\triangle AST$: $\cos \angle CAT = \frac{AS}{AT}$

$$\cos \angle CAT = \frac{\frac{1}{2}\sqrt{50}}{6}$$
$$\angle CAT \approx 53{,}9°$$

b In $\triangle APQ$: $PQ^2 = AP^2 + AQ^2 - 2 \cdot AP \cdot AQ \cdot \cos \angle A$
$= 16 + 36 - 2 \cdot 4 \cdot 6 \cdot \cos 53{,}9°$
$\approx 23{,}72$
$PQ \approx 4{,}87$

c In $\triangle ABQ$: $BQ^2 = AB^2 + AQ^2 - 2 \cdot AB \cdot AQ \cdot \cos \angle A$
$= 25 + 36 - 2 \cdot 5 \cdot 6 \cdot \cos 45°$
$\approx 18{,}57$
$BQ \approx 4{,}31$

21



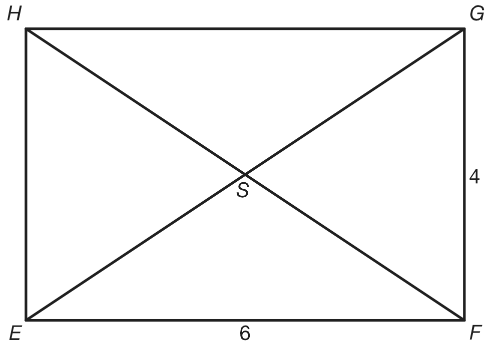
In $\triangle ABS$ is $AB = 6$, $AS = 4$ en $BS = 2\frac{1}{2}$
De cosinusregel geeft
$AB^2 = AS^2 + BS^2 - 2 \cdot AS \cdot BS \cdot \cos \angle ASB$
$36 = 16 + 6\frac{1}{4} - 2 \cdot 4 \cdot 2\frac{1}{2} \cdot \cos \angle ASB$
$36 = 22\frac{1}{4} - 20 \cos \angle ASB$
$20 \cos \angle ASB = -13\frac{3}{4}$
$\cos \angle ASB = \dfrac{-13\frac{3}{4}}{20}$
$\angle ASB \approx 133{,}4°$
$\angle BSC = 180° - \angle ASB \approx 46{,}6°$
In $\triangle BCS$ geeft de cosinusregel $BC^2 = BS^2 + CS^2 - 2 \cdot BS \cdot CS \cdot \cos \angle BSC$
$= 6\frac{1}{4} + 16 - 2 \cdot 2\frac{1}{2} \cdot 4 \cdot \cos 46{,}6°$
$\approx 8{,}5$
$BC \approx 2{,}92$
De omtrek $= 2 \cdot AB + 2 \cdot BC = 2 \cdot 6 + 2 \cdot 2{,}92 \approx 17{,}83$
In $\triangle ABS$: $AS^2 = AB^2 + BS^2 - 2 \cdot AB \cdot BS \cdot \cos \angle ABS$
$16 = 36 + 6\frac{1}{4} - 2 \cdot 6 \cdot 2\frac{1}{2} \cdot \cos \angle ABS$
$16 = 42\frac{1}{4} - 30 \cos \angle ABS$
$30 \cos \angle ABS = 26\frac{1}{4}$
$\cos \angle ABS = \dfrac{26\frac{1}{4}}{30}$
$\angle ABS \approx 29{,}0°$
Noem in $\triangle ABD$ de hoogte vanuit D op AB h.
dan geldt $\sin \angle ABS = \dfrac{h}{5}$, dus $h = 5 \cdot \sin 29{,}0° \approx 2{,}42$
Dit geeft $O(ABCD) \approx 6 \cdot 2{,}42 = 14{,}52$
Alternatieve manier voor het berekenen van de oppervlakte.
De diagonalen verdelen het parallellogram in vier driehoeken met gelijke oppervlakte.
$O(ABCD) = 4 \cdot O(\triangle ABS) = 4 \cdot \frac{1}{2} \cdot SA \cdot SB \cdot \sin \angle ASB$
$= 4 \cdot \frac{1}{2} \cdot 4 \cdot 2\frac{1}{2} \cdot \sin 133{,}43° \approx 14{,}53$

22 In $\triangle PAC$: $\tan \angle APC = \dfrac{AC}{AP}$
$\tan 10° = \dfrac{60}{AP}$
$AP = \dfrac{60}{\tan 10°} \approx 340{,}3 \text{ m}$
In $\triangle BPD$: $\tan \angle BPD = \dfrac{BD}{BP}$
$\tan 5° = \dfrac{46}{BP}$
$BP = \dfrac{46}{\tan 5°} \approx 525{,}8 \text{ m}$
In $\triangle ABP$: $AB^2 = AP^2 + BP^2 - 2 \cdot AP \cdot BP \cdot \cos \angle APB$
$= 340{,}3^2 + 525{,}8^2 - 2 \cdot 240{,}3 \cdot 525{,}8 \cdot \cos 40°$
$\approx 118\,127{,}0$
$AB \approx 344 \text{ m}$

6.3 Hoeken bij lijnen en vlakken

bladzijde 79

23 **a**



b In $\triangle HFG$: $\tan\angle HFG = \frac{HG}{FG}$

$\tan\angle HFG = \frac{6}{4}$

$\angle HFG \approx 56{,}3°$

$\triangle SFG$ is gelijkbenig.

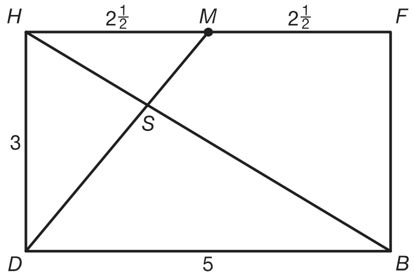
$\angle FSG = 180° - 2 \cdot 56{,}3° \approx 67°$

c $\angle ESF = 180° - \angle FSG \approx 180° - 67° \approx 113°$

bladzijde 80

24 **a** Omdat de hoek tussen twee snijdende lijnen een niet-stompe hoek is, is deze hoek hoogstens 90°.
Mathilde's antwoord is dus fout.
Wél goed is: $\angle(BH, DM) \approx 180° - 99° \approx 81°$

b



Met de stelling van Pythagoras in $\triangle DMH$: $DM = \sqrt{3^2 + \left(2\frac{1}{2}\right)^2} = \sqrt{15{,}25}$.

Met de stelling van Pythagoras in $\triangle$DBH: BH $= \sqrt{3^2 + 5^2} = \sqrt{34}$.

$\triangle DBS \backsim \triangle MHS$ (zandloperfiguur)

Omdat $DB : MH = 2 : 1$, is ook $DS : MS = BS : HS = 2 : 1$.

Dit geeft $DS = \frac{2}{3}DM = \frac{2}{3}\sqrt{15{,}25}$ en HS $= \frac{1}{3}$BH $= \frac{1}{3}\sqrt{34}$.

Cosinusregel in $\triangle DSH$:

$DH^2 = DS^2 + HS^2 - 2 \cdot DS \cdot HS \cdot \cos\angle DSH$

$3^2 = \left(\frac{2}{3}\sqrt{15{,}25}\right)^2 + \left(\frac{1}{3}\sqrt{34}\right)^2 - 2 \cdot \frac{2}{3}\sqrt{15{,}2} \cdot \frac{1}{3}\sqrt{34} \cdot \cos\angle \text{DSH}$

$9 = 6\frac{7}{9} + 3\frac{7}{9} - \frac{4}{9}\sqrt{518{,}5} \cdot \cos\angle DSH$

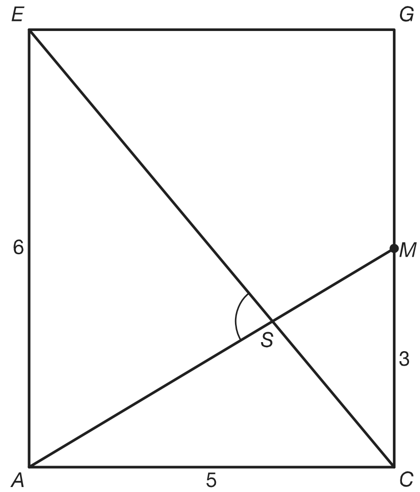
$-1\frac{5}{9} = -\frac{4}{9}\sqrt{518{,}5} \cdot \cos\angle DSH$

$\cos\angle DSH \approx 0{,}1537$

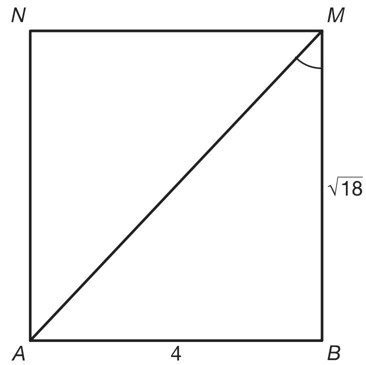
$\angle DSH \approx 81°$

De methode van het voorbeeld is handiger.

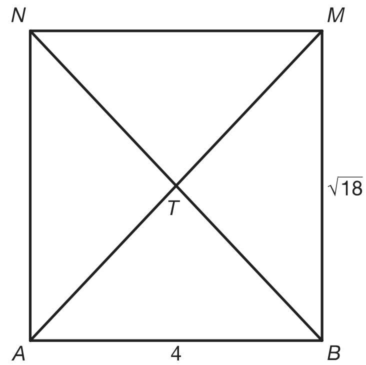
25 **a** De stelling van Pythagoras in
$\triangle ABC$: $AC = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$.
In ΔACE: $\tan \angle ACE = \frac{AE}{AC} = \frac{6}{5}$
geeft $\angle ACE \approx 50{,}2°$.
In $\triangle ACM$: $\tan \angle CAM = \frac{CM}{AC} = \frac{3}{5}$
geeft $\angle CAM \approx 31{,}0°$.
In $\triangle ACS$: $\angle ASC \approx 180° - 50{,}2° - 31{,}0 \approx 98{,}8°$.
Dan is $\angle ASE \approx 180° - 98{,}8° \approx 82{,}2°$.
Dus $\angle(AM, CE) \approx 82°$.



b De stelling van Pythagoras in
$\triangle BCM$: $BM = \sqrt{3^2 + 3^2} = \sqrt{18}$.
$\angle(AM, BM) = \angle AMB$
In $\triangle AMB$: $\tan \angle AMB = \frac{AB}{BM} = \frac{4}{\sqrt{18}}$
geeft $\angle AMB \approx 43{,}3°$.
Dus $\angle(AM, BM) \approx 43°$.



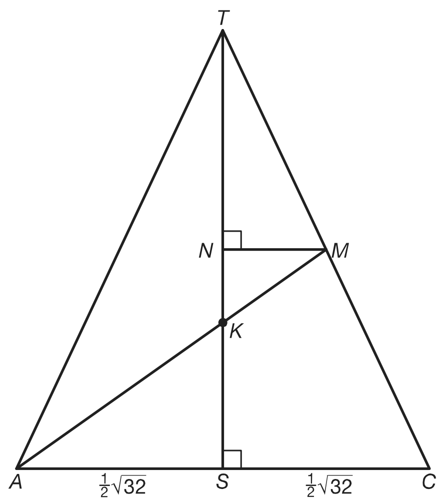
c In $\triangle BTM$: $\angle BTM \approx 180° - 2 \cdot 43{,}3° = 93{,}4°$.
Dan is $\angle ATB \approx 180° - 93{,}4° = 86{,}6°$.
Dus $\angle(AM, BN) \approx 87°$.



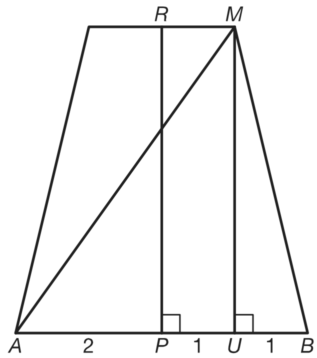
bladzijde 81

26 **a** De stelling van Pythagoras in
$\triangle ABC$: $AC = \sqrt{4^2 + 4^2} = \sqrt{32}$.
Dan is $AS = SC = \frac{1}{2}\sqrt{32}$.
Teken hulplijn MN met N op TS en $MN \perp TS$.
$\triangle ASK \backsim \triangle MNK$ (zandloperfiguur)

Omdat $AS : MN = 2 : 1$, is ook $KS : KN = 2 : 1$.
$$\left.\begin{array}{l} KS = \frac{2}{3}NS \\ NS = \frac{1}{2}TS \end{array}\right\} KS = \frac{2}{3} \cdot \frac{1}{2}TS = \frac{1}{3} \cdot TS = \frac{1}{3} \cdot 6 = 2$$
In $\triangle AKS$: $\tan \angle AKS = \frac{AS}{KS} = \frac{\frac{1}{2}\sqrt{32}}{2}$
geeft $\angle AKS \approx 54{,}7°$
Dus $\angle(AM, TS) \approx 55°$.
In plaats van de hulplijn MN met N op TS had je ook
de hulplijn MN met N op AC (en $MN \perp AC$) kunnen tekenen.

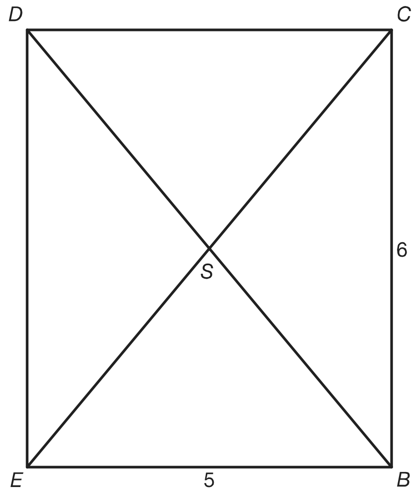


b P is het midden van AB, Q het midden van CD en R het midden van QT.
Het punt L ligt op PQ recht onder R.
De stelling van Pythagoras in $\triangle PLR$: $PR = \sqrt{3^2 + 3^2} = \sqrt{18}$.
Hiernaast is de doorsnede van ABM met de piramide apart getekend.



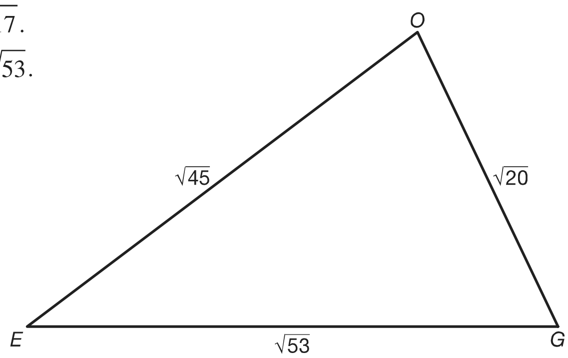
In $\triangle AMU$: $\tan\angle AMU = \dfrac{AU}{MU} = \dfrac{3}{\sqrt{18}}$ geeft $\angle AMU \approx 35{,}3°$.
In $\triangle BMU$: $\tan\angle BMU = \dfrac{BU}{MU} = \dfrac{1}{\sqrt{18}}$ geeft $\angle BMU \approx 13{,}3°$.
$\angle AMB = \angle AMU + \angle BMU \approx 35{,}3° + 13{,}3° \approx 48{,}5°$.
Dus is $\angle(AM, BM) \approx 49°$.

27 **a** De stelling van Pythagoras in
$\triangle OCD: CD = \sqrt{4^2 + 3^2} = 5$.
In $\triangle BEC$: $\tan\angle BEC = \dfrac{BC}{BE} = \dfrac{6}{5}$ geeft $\angle BEC \approx 50{,}2°$.
Dan is $\angle ESB = 180° - 2 \cdot 50{,}2° = 79{,}6°$.
Dus $\angle(BD, CE) \approx 80°$.



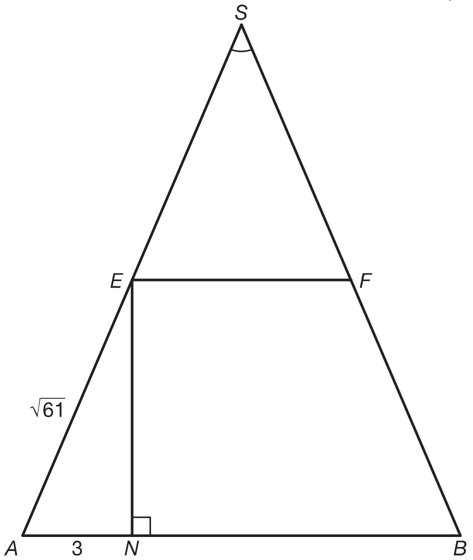
b De stelling van Pythagoras in $\triangle OCG$: $OG = \sqrt{4^2 + 2^2} = \sqrt{20}$.
De stelling van Pythagoras in $\triangle OAE$: $OE = \sqrt{6^2 + 3^2} = \sqrt{45}$.
Het punt H is het punt (0, 0, 2).
De stelling van Pythagoras in $\triangle HDG$: $DG = \sqrt{4^2 + 1^2} = \sqrt{17}$.
De stelling van Pythagoras in $\triangle EDG$: $EG = \sqrt{6^2 + 17^2} = \sqrt{53}$.

cosinusregel in $\triangle EGO$:
$OE^2 = OG^2 + EG^2 - 2 \cdot OG \cdot EG \cdot \cos\angle EGO$
$45 = 20 + 53 - 2 \cdot \sqrt{20} \cdot \sqrt{53} \cdot \cos\angle EGO$
$-28 = -2 \cdot \sqrt{20} \cdot \sqrt{53} \cdot \cos\angle EGO$
$\cos\angle EGO = \dfrac{28}{2 \cdot \sqrt{20} \cdot \sqrt{53}} \approx 0{,}43$
$\angle EGO \approx 64{,}5°$
Dus $\angle(OG, EG) \approx 65°$.



28 **a** M is het midden van AD.
De stelling van Pythagoras in $\triangle ME'E$: $ME = \sqrt{3^2 + 6^2} = \sqrt{45}$.
In ΔMAE: $\tan\angle MAE = \dfrac{EM}{AM} = \dfrac{\sqrt{45}}{4}$ geeft $\angle MAE \approx 59{,}2°$.
Omdat $\triangle ADE$ gelijkbenig is, is $\angle AED = 180° - 2 \cdot 59{,}2° \approx 61{,}6°$.
Dus $\angle(AE, DE) \approx 62°$.

b De stelling van Pythagoras in $\triangle$AME: $AE = \sqrt{4^2 + \sqrt{45}^2} = \sqrt{61}$.



$\angle(AE, BF) = \angle ASB = 2 \cdot \angle AEN$

In $\triangle$AEN: $\sin\angle AEN = \dfrac{3}{\sqrt{61}}$ geeft $\angle AEN \approx 22{,}6°$.

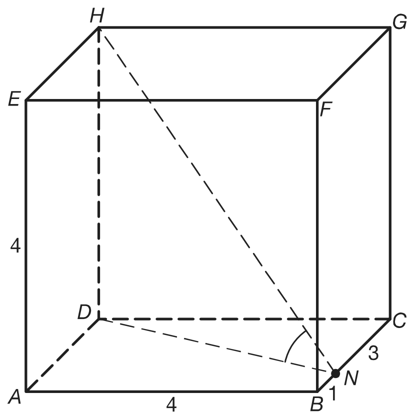
Dus is $\angle(AC, BF) \approx 2 \cdot 22{,}6 \approx 45°$.

c $\angle AEF = \angle AEN + \angle NEF \approx 45° + 90° \approx 135°$.

29 De tangens van de gevraagde hoek is $\dfrac{50}{40}$, dus de hoek is ongeveer 51°.

bladzijde 83

30 **a**



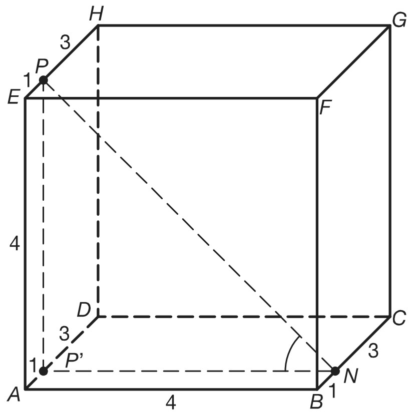
b $\angle(HN, ABC) = \angle(HN, DN) = \angle DNH$

In $\triangle DCN$ is $DN = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$.

In $\triangle DNH$: $\tan\angle DNH = \dfrac{DH}{DN} = \dfrac{4}{5}$ geeft $\angle DNH \approx 38{,}7°$.

Dus de hellingshoek van HN is ongeveer 39°.

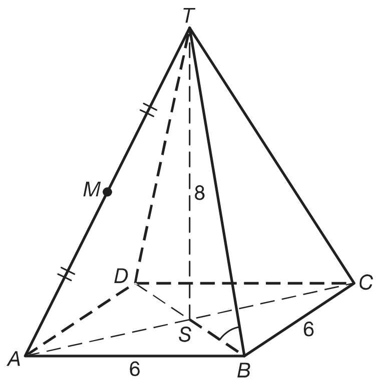
c De kleinste hoek krijg je als $P = H$. Dan is $\angle(PN, ABC) = \angle DNH \approx 38{,}7°$ (zie vraag a).
De grootste hoek krijg je als $EP = 1$. Zie de figuur hieronder.



De projectie van P op ABC is P'. $\triangle PNP'$ is gelijkbenig en rechthoekig.
De grootste hoek is dus 45°.
Dus kan $\angle(PN, ABC)$ waarden aannemen tussen 38,7° en 45°.

31 **a** $\angle(TS, ABC) = 90°$ en $\angle(AC, BDT) = 90°$.

b

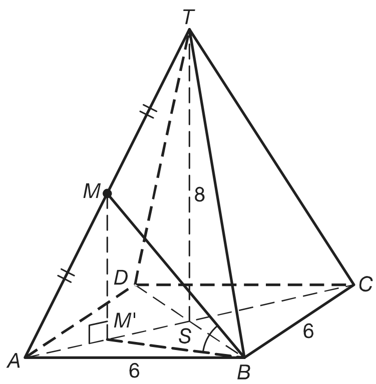


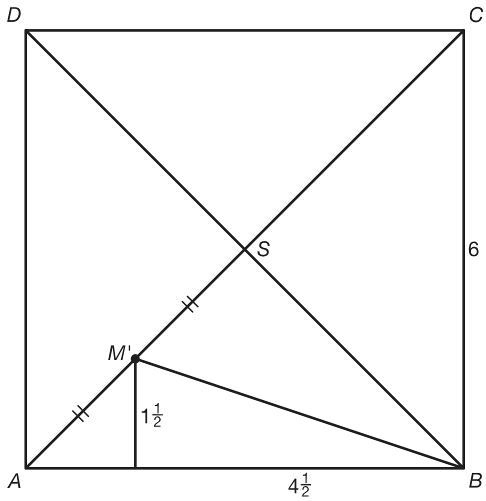
$\angle(BT, ABC) = \angle(BT, SB) = \angle TBS$

$\text{BS} = \frac{1}{2}\text{BD} = \frac{1}{2} \cdot \sqrt{6^2 + 6^2} = \frac{1}{2}\sqrt{72}$

In $\triangle$TBS: $\tan\angle\text{TBS} = \frac{\text{TS}}{\text{BS}} = \frac{8}{\frac{1}{2}\sqrt{72}}$ geeft $\angle TBS \approx 62°$.

c $\angle(BM, ABC) = \angle(BM, BM') = \angle MBM'$



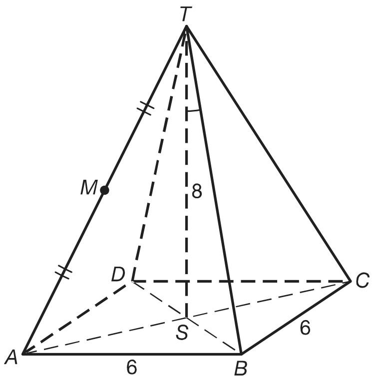


$\text{M'B} = \sqrt{(4\frac{1}{2})^2 + (1\frac{1}{2})^2} = \sqrt{22{,}5}$

In $\triangle MBM'$: $\tan\angle MBM' = \frac{MM'}{M'B} = \frac{4}{\sqrt{22{,}5}}$ geeft $\angle MBM' \approx 40°$.

Dus $\angle(BM, ABC) \approx 40°$.

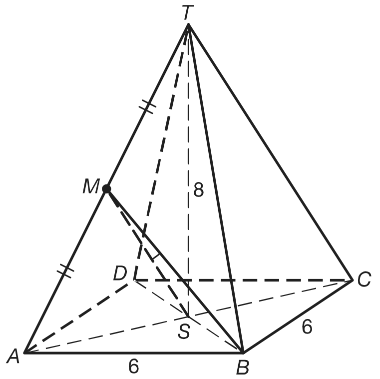
d



$\angle(BT, ACT) = \angle(BT, ST) = \angle BTS$

In $\triangle$BTS: $\tan\angle \text{BTS} = \dfrac{\text{BS}}{\text{ST}} = \dfrac{\frac{1}{2}\sqrt{72}}{8}$ geeft $\angle BTS \approx 28°$.

e



$\angle(BM, ACT) = \angle(BM, SM) = \angle BMS$

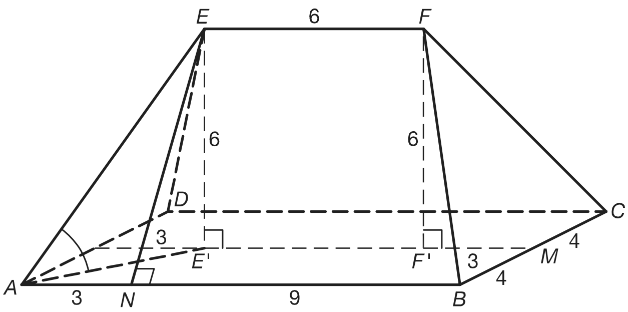
M' is midden van *AS*.

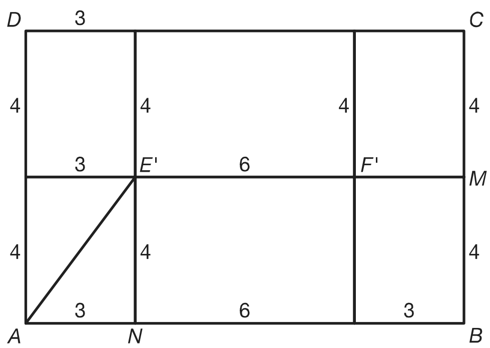
$\text{MS} = \sqrt{\text{M'S}^2 + \text{MM'}^2} = \sqrt{\left(\frac{1}{4}\sqrt{72}\right)^2 + 4^2} = \sqrt{20{,}5}$

In $\triangle$BMS: $\tan\angle\text{BMS} = \dfrac{\text{BS}}{\text{MS}} = \dfrac{\frac{1}{2}\sqrt{72}}{\sqrt{20{,}5}}$ geeft $\angle BMS \approx 43°$.

Dus $\angle(BM, ACT) \approx 43°$.

32 **a**





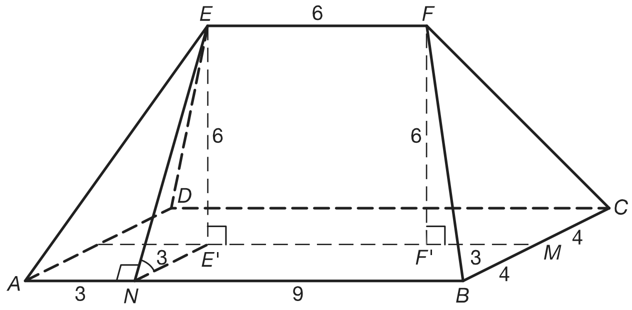
$\angle(AE, ABC) = \angle(AE, AE') = \angle EAE'$

$AE' = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

In $\triangle EAE'$: $\tan \angle EAE' = \dfrac{EE'}{AE'} = \dfrac{6}{5}$ geeft $\angle EAE' \approx 50°$.

Dus $\angle(AE, ABC) \approx 50°$.

b

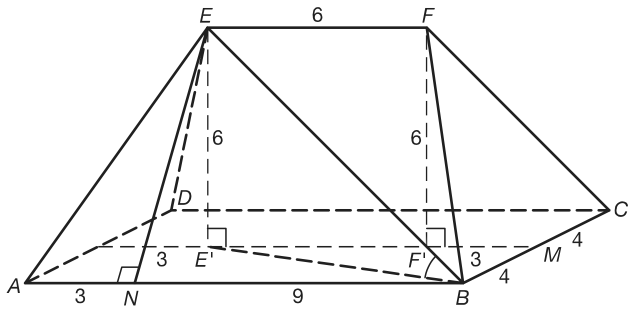


$\angle(NE, ABC) = \angle(NE, NE') = \angle ENE'$

In $\triangle ENE'$: $\tan \angle ENE' = \dfrac{EE'}{NE'} = \dfrac{6}{4}$ geeft $\angle ENE' \approx 56°$.

Dus $\angle(NE, ABC) \approx 56°$.

c



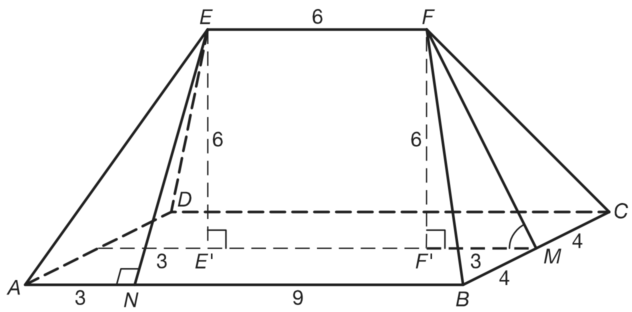
$\angle(BE, ABC) = \angle(BE, BE') = \angle EBE'$

$BE' = \sqrt{9^2 + 4^2} = \sqrt{97}$

In $\triangle EBE'$: $\tan \angle EBE' = \dfrac{EE'}{BE'} = \dfrac{6}{\sqrt{97}}$ geeft $\angle EBE' \approx 31°$.

Dus $\angle(BE, ABC) \approx 31°$.

d



$\angle(MF, ABC) = \angle MF, MF') = \angle FMF'$

In $\Delta FMF'$: $\tan \angle FMF' = \dfrac{FF'}{MF'} = \dfrac{6}{3} = 2$ geeft $\angle FMF' \approx 63°$.

Dus $\angle(MF, ABC) \approx 63°$.

33 **a** De hellingshoek van het dakdeel $ABFE$ is $\angle(NE, ABC)$.

b De hellingshoek van het dakdeel BCF is $\angle(MF, ABC)$.

bladzijde 86

34 **a** De vlakken ACH en ABC snijden elkaar volgens de lijn AC.
Een standvlak is het vlak door DH loodrecht op AC.

$AC = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

In $\triangle ACD$ geeft de zijde $\times$ hoogte-methode

$AC \times DD' = AD \times CD$

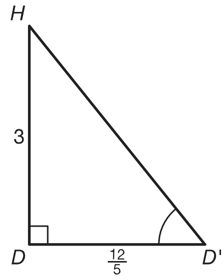
$5 \times DD' = 3 \times 4$

$DD' = \frac{12}{5}$

In $\triangle HD'D$: $\tan \angle HD'D = \dfrac{HD}{DD'} = \dfrac{3}{\frac{12}{5}}$ geeft $\angle HD'D \approx 51°$.

Dus $\angle(ACH, ABC) \approx 51°$.

b De vlakken BMG en ABC snijden elkaar volgens de lijn BM.
Een standvlak is het vlak door CG loodrecht op BM.



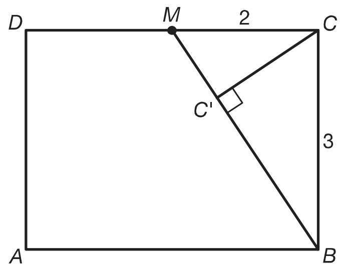
$BM = \sqrt{3^2 + 2^2} = \sqrt{13}$

In $\triangle BCM$ geeft de zijde $\times$ hoogte-methode

$BM \times CC' = MC \times BC$

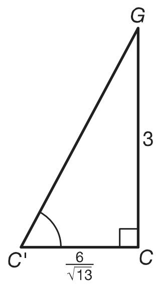
$\sqrt{13} \times CC' = 2 \times 3$

$CC' = \dfrac{6}{\sqrt{13}}$

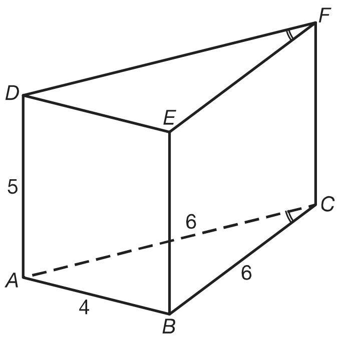


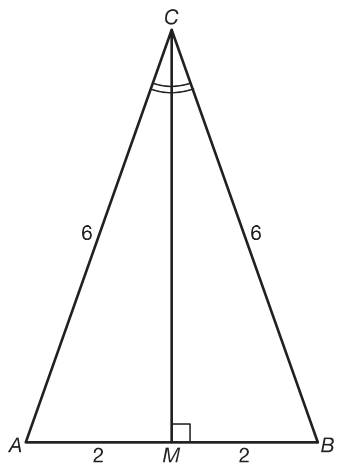
In $\Delta GC'C$: $\tan \angle GC'C = \dfrac{GC}{CC'} = \dfrac{3}{\frac{6}{\sqrt{13}}}$ geeft $\angle GC'C \approx 61°$.

Dus $\angle(BMG, ABC) \approx 61°$.



35 **a**





$\angle(ACF, BCF) = \angle(AC, BC) = \angle ACB$

$\angle ACB = 2 \cdot \angle ACM$

$\sin \angle ACM = \dfrac{2}{6}$

$\angle ACM \approx 19{,}5°$

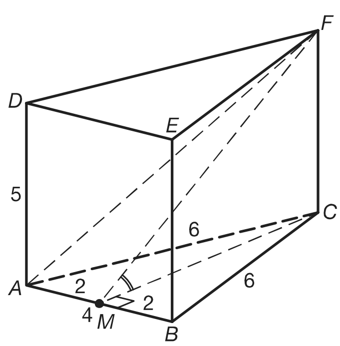
Dus $\angle(ACF, BCF) = 2 \cdot \angle ACM \approx 39°$.

b $\angle(ABF, ABC) = \angle FMC$

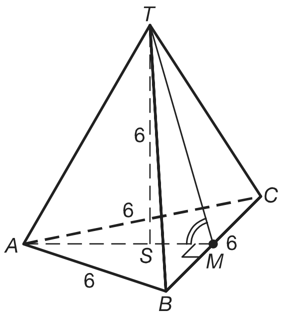
$\tan \angle FMC = \dfrac{FC}{MC} = \dfrac{5}{\sqrt{6^2 - 2^2}} = \dfrac{5}{\sqrt{32}}$

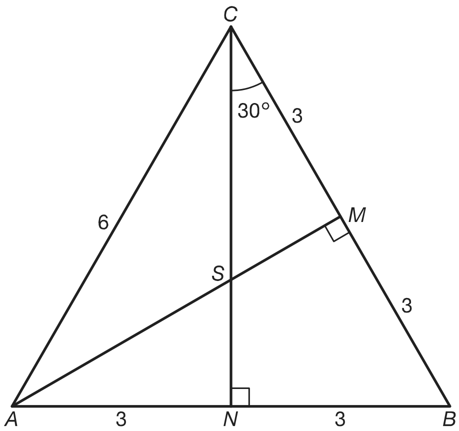
$\angle FMC \approx 41{,}47°$

Dus $\angle(ABF, ABC) \approx 41°$.



36





$\angle(BCT, ABC) = \angle AMT$

$\triangle ABC$ is een gelijkzijdige driehoek.

In $\triangle CMS$ is $\tan 30° = \dfrac{SM}{MC}$, dus $SM = MC \cdot \tan 30° = 3 \cdot \tan 30° \approx 1{,}73$.

In $\triangle SMT$: $\tan\angle M = \dfrac{ST}{SM} = \dfrac{6}{1{,}73}$ geeft $\angle M \approx 73{,}9°$.

Dus $\angle(BCT, ABC) \approx 74°$.

37 **a** De hoeken die de opstaande zijvlakken met elkaar maken zijn de hoeken van $\triangle ABC$.

$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos\angle A$

$25 = 36 + 49 - 2 \cdot 6 \cdot 7 \cdot \cos\angle A$

$84\cos\angle A = 60$

$\cos\angle A = \dfrac{60}{84}$

$\angle A \approx 44{,}42°$

$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos\angle B$

$49 = 36 + 25 - 2 \cdot 6 \cdot 5 \cdot \cos\angle B$

$60\cos\angle B = 12$

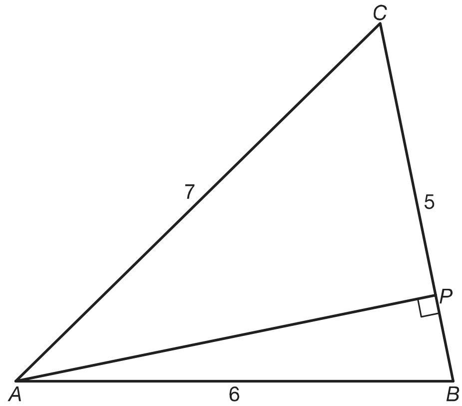
$\cos\angle B = \dfrac{12}{60}$

$\angle B \approx 78{,}46°$

$\angle C = 180° - \angle A - \angle B$

$\approx 180° - 44{,}42° - 78{,}46° \approx 57{,}12°$

Dus $\angle(ABE, ACF) \approx 44°$, $\angle(ABE, BCF) \approx 78°$
en $\angle(ACF, BCF) \approx 57°$.



b Zie de figuur bij a.

De hellingshoek van vlak $BCD = \angle(DP, AP) = \angle DPA$.

In $\triangle ABP$: $\sin\angle B = \dfrac{AP}{AB}$ dus $AP = AB \cdot \sin\angle B \approx 6 \cdot \sin 78{,}46° \approx 5{,}88$.

$\tan\angle DPA = \dfrac{AD}{AP} \approx \dfrac{4}{5{,}88}$

$\angle DPA \approx 34{,}23°$

Dus de hellingshoek van vlak BCD is 34°.

c De hellingshoek van vlak $BDF = \angle(BQ, QE)$ met Q op DF zo, dat $EQ \perp DF$.

$\angle D = \angle BAC \approx 44{,}42°$

$\sin\angle D = \dfrac{EQ}{DE}$

$EQ = DE \cdot \sin\angle D$

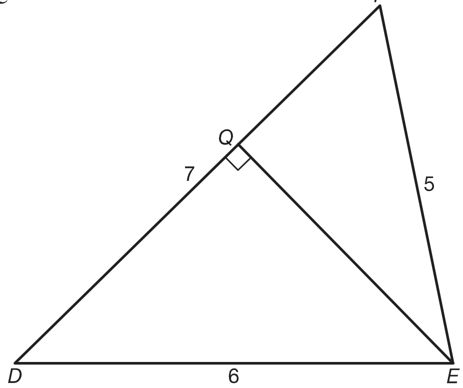
$EQ \approx 6 \cdot \sin 44{,}42°$

$EQ \approx 4{,}20$

$\tan\angle BQE = \dfrac{BE}{EQ} \approx \dfrac{4}{4{,}20}$

$\angle BQE \approx 43{,}61°$

Dus de hellingshoek van vlak BDF is 44°.

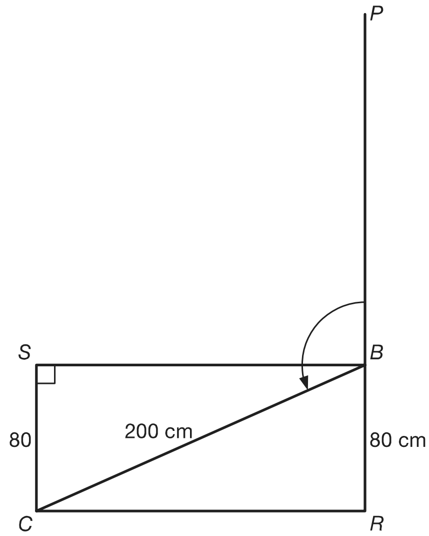


38 **a** De evenwijdige vlakken V en W hebben geen standvlak want evenwijdige vlakken snijden elkaar niet.
b $\angle(V,W) = 0°$.

6.4 Rotaties

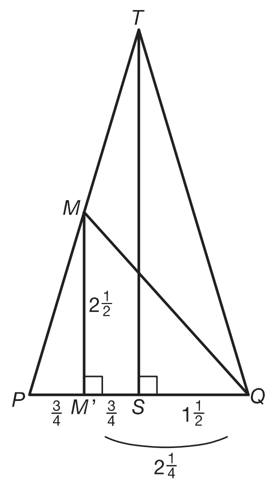
bladzijde 88

39 $\sin \angle CBS = \frac{CS}{BC} = \frac{80}{200}$
$\angle CBS \approx 23{,}58°$
De de rotatiehoek is $90° + \angle CBS \approx 114°$.



bladzijde 89

40 **a** A beschrijft een cirkelboog met straal $AB = 3$ dm en middelpuntshoek $\alpha \approx 107°$.
Lengte baan = $\frac{107}{360} \cdot 2\pi \cdot 3 \approx 5{,}6$ dm.
b $MM' = \frac{1}{2}TS = 2\frac{1}{2}$
In $\triangle M'QM$ geeft de stelling van Pythagoras
$MQ^2 = \left(2\frac{1}{2}\right)^2 + \left(2\frac{1}{4}\right)^2 = 11{,}3125.$
$MQ = \sqrt{11{,}3125} \approx 3{,}36$
M beschrijft een cirkelboog met straal $MQ \approx 3{,}36$.
Lengte baan = $\frac{107}{360} \cdot 2\pi \cdot 3{,}36 \approx 6{,}3$ dm.
c De lijn NM is evenwijdig aan de rotatieas.
Alle punten op deze lijn leggen dus een even grote afstand af bij deze rotatie.



bladzijde 90

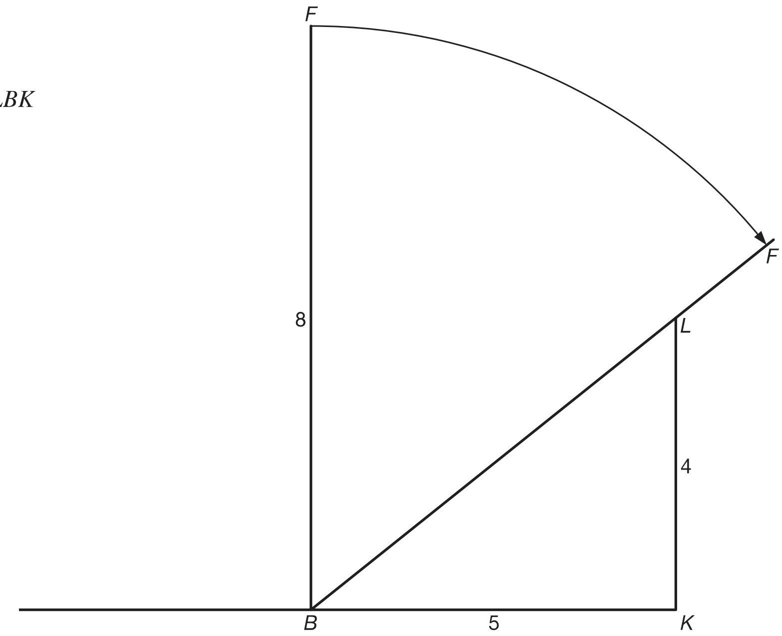
41 **a** De rotatiehoek is 90°.
De baan van F is een kwart cirkel met straal $BF = 8$ dm.
Lengte baan = $\frac{1}{4} \cdot 2\pi \cdot 8 \approx 12{,}6$ dm.
b $BE = \sqrt{4^2 + 8^2} = \sqrt{80}$
De baan van E is een kwart cirkel met straal $BE = \sqrt{80}$ dm.
Lengte baan = $\frac{1}{4} \cdot$ 2Ó$\cdot \sqrt{80} \approx 14{,}0$ dm.
c $BM = \sqrt{4^2 + 4^2} = \sqrt{32}$
De baan van M is een kwart cirkel met straal $BM = \sqrt{32}$ dm.
Lengte baan = $\frac{1}{4} \cdot$ 2Ó$\cdot \sqrt{32} \approx 8{,}9$ dm.

42 **a** $\tan \angle LBK = \dfrac{4}{5}$

$\angle LBK \approx 38{,}7°$

De rotatiehoek is $\angle FBF' = 90° - \angle LBK$

$\approx 90° - 38{,}7° = 51{,}3°.$

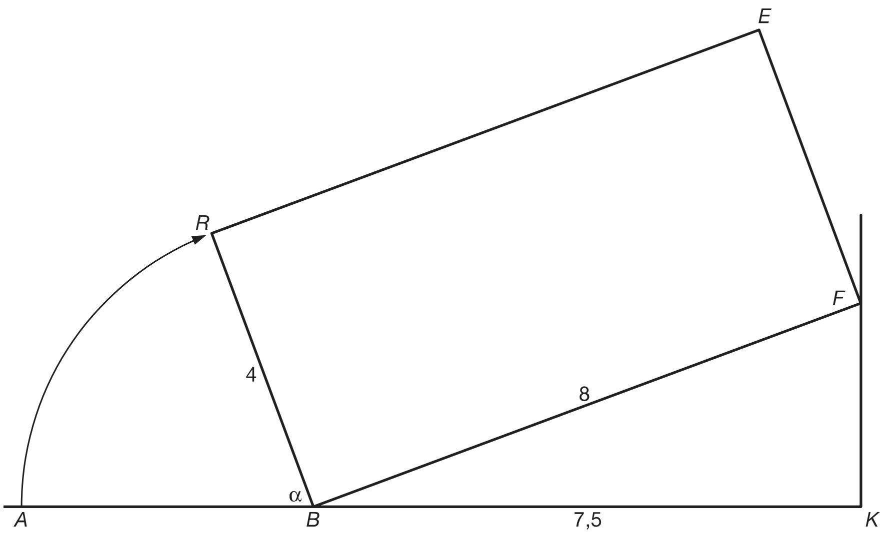


b $CH = BE = \sqrt{80}$

De baan van H is een cirkelboog met straal $\sqrt{80}$.

Lengte baan $= \dfrac{51{,}3}{360} \cdot 2\pi \cdot \sqrt{80} \approx 8{,}0$ dm.

c



De kist komt nu tegen de zijkant van de muur.

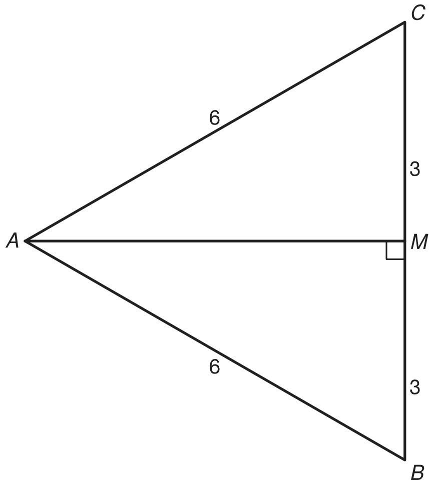
$\cos \angle FBK = \dfrac{7{,}5}{8}$

$\angle FBK \approx 20{,}4°$

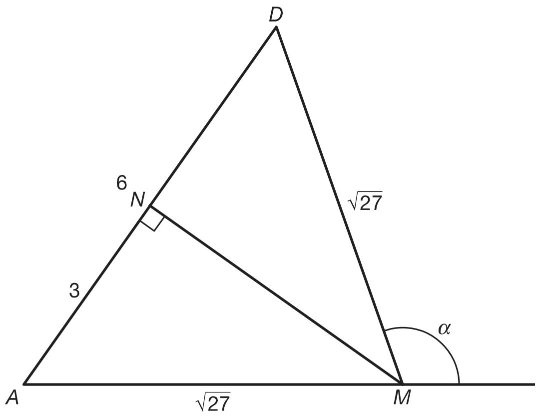
De rotatiehoek α is $180° - 90° - \angle FBK \approx 90° - 20{,}4 = 69{,}6°.$

d Lengte baan $= \dfrac{69{,}6}{360} \cdot 2\pi \cdot \sqrt{80} \approx 10{,}9$ dm.

43 **a** Het vlak door A, D en het midden van M van BC staat loodrecht op BC.



$AM = \sqrt{6^2 - 3^2} = \sqrt{27}$. Ook $DM = \sqrt{27}$.



In $\triangle AMN$: $\sin \angle A = \dfrac{3}{\sqrt{27}}$

$\angle AMN \approx 35{,}3°$

$\angle AMD = 2 \cdot \angle AMN \approx 70{,}5°$

$\alpha = 180° - \angle AMD \approx 109{,}47°$

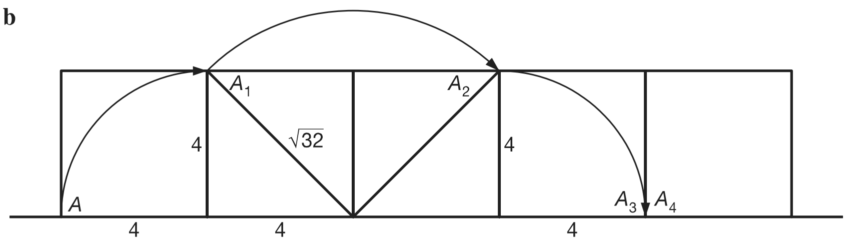
Dus de rotatiehoek is 109°.

b A beschrijft een cirkelboog met straal $MA = \sqrt{27}$.

Lengte baan $= \dfrac{109}{360} \cdot 2\pi \cdot \sqrt{27} \approx 9{,}9$ dm.

bladzijde 91

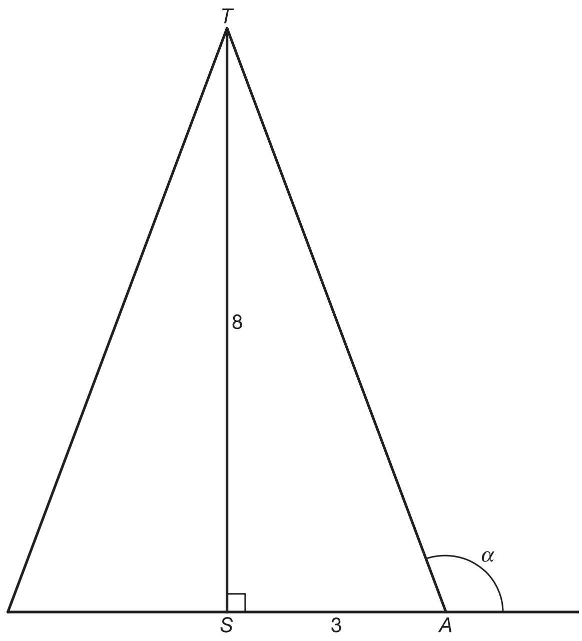
44 **a, b**



c Lengte baan = 2 · kwart cirkel met straal 4 dm + kwart cirkel met straal $\sqrt{32}$ dm

$= 2 \cdot \frac{1}{4} \cdot 2\pi \cdot 4 + \frac{1}{4} \cdot 2\pi \cdot \sqrt{32} = 4\pi + \frac{1}{2}\pi\sqrt{32} \approx 21{,}5$ dm.

45 **a**



In $\triangle SAT$: $\tan \angle A = \frac{8}{3}$

$\angle A \approx 69{,}4°$

$\alpha = 180° - \angle A \approx 110{,}6°$

Verder is $AT = \sqrt{3^2 + 8^2} = \sqrt{73}$.
T beschrijft een cirkelboog met straal AT,
dus lengte baan $= \frac{110{,}6}{360} \cdot 2\pi \cdot \sqrt{73} \approx 16{,}5$ dm.

b Als Bas gelijk heeft, dan is de omtrek van de cirkel met straal AT een geheel aantal keren de omtrek van de grondcirkel van de kegel.

Omtrek van cirkel met straal AT is $2\pi \cdot \sqrt{73}$.
Omtrek van grondcirkel kegel $= 2\pi \cdot 3$.

$\frac{2\pi \cdot \sqrt{73}}{2\pi \cdot 3} \approx 2{,}848$ en is dus niet een geheel getal.
Bas heeft dus geen gelijk.

bladzijde 92

46 **a** $h = 2 \cdot 49 = 98$
De hoogte van de stoel is dus 98 cm.

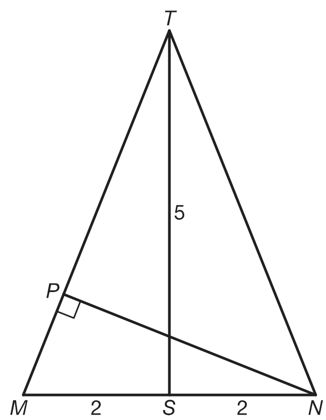
b Bij de trap bevindt B zich recht boven T.
De hoogte van de trap is dus ook 98 cm.

c De rotatiehoek is 180°.
De straal van de baan is $ST = \sqrt{28^2 + 49^2} = \sqrt{3185}$.
De lengte van de baan is $\frac{180}{360} \cdot$ 2Ó· $\sqrt{3185} \approx 177{,}3 \approx 177$ cm.

47 **a** Ja, dit is juist.

b Ook dit klopt want de straal van de cirkelboog is niet overal hetzelfde, zo is bij A de straal 4 en bij T is de straal $\sqrt{2^2 + 5^2} = \sqrt{29}$.

c M is het midden van AD en N is het midden van BC.
Het getekende punt P is één van de punten met minimale baanlengte.
De andere punten van het vlak ADT met dezelfde baanlengte liggen op de lijn door P die evenwijdig aan AD is.

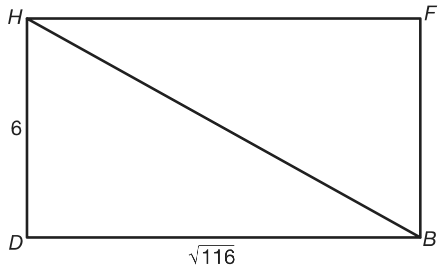


6.5 Afstanden in de ruimte

bladzijde 94

48 **a** $BD = \sqrt{10^2 + 4^2} = \sqrt{116}$

$BH = \sqrt{116 + 6^2} = \sqrt{152} \approx 12{,}33$



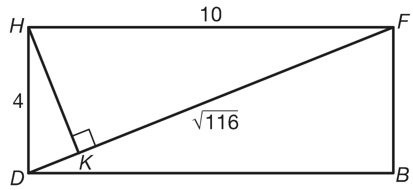
b In $\triangle EGK$ zijde × hoogte-methode

$EG \times HK = EH \times HG$

$\sqrt{116} \times HK = 4 \times 10$

$HK = \dfrac{40}{\sqrt{116}} \approx 3{,}71$

De afstand van H tot EG is 3,71.



c De afstand van H tot ABC is de hoogte van de balk en is dus 6.

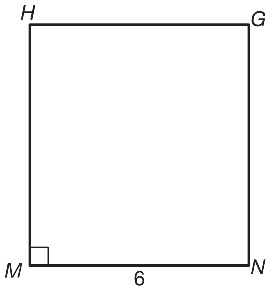
d De afstand van AH tot BG is gelijk aan de afstand tussen de vlakken $ADHE$ en $BCGF$ en is dus 10.

e De afstand van EG tot ABC is de hoogte van de balk en is dus 6.

bladzijde 96

49 **a** $d(E, BDH) = d(E, HF) = \frac{1}{2} \cdot \text{EG} = \frac{1}{2} \cdot \sqrt{6^2 + 6^2} = \frac{1}{2}\sqrt{72} \approx 4{,}24$

b



$d(H, MN) = HM = \sqrt{6^2 + 3^2} = \sqrt{45} \approx 6{,}71$

c $d(N, EG) = NN'$

$EN = \sqrt{6^2 + 3^2} = \sqrt{45}$

$GN = \sqrt{45}$

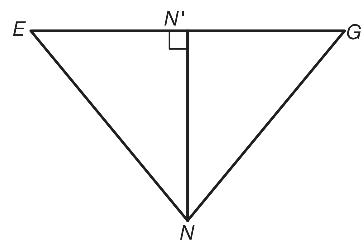
$EG = \sqrt{72}$ dus $EN' = \frac{1}{2}\sqrt{72}$

In $\triangle ENN'$ geldt $\left(\frac{1}{2}\sqrt{72}\right)^2 + (NN')^2 = \left(\sqrt{45}\right)^2$

dus $(NN')^2 = 45 - \frac{1}{4} \cdot 72 = 27$

$NN' = \sqrt{27}$

Dus $d(N, EG) = \sqrt{27} \approx 5{,}20$.

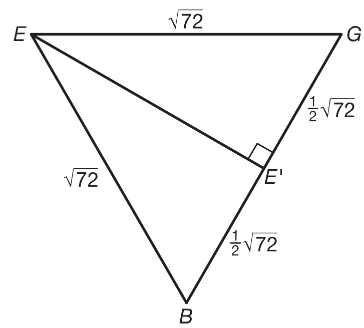


d In $\triangle GEE'$ geldt $(EE')^2 + \left(\frac{1}{2}\sqrt{72}\right)^2 = \left(\sqrt{72}\right)^2$

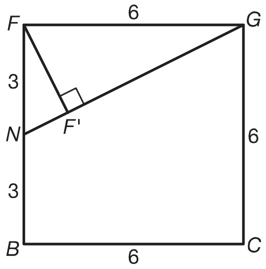
$(EE')^2 = 72 - \frac{1}{4} \cdot 72 = 54$

$(EE') = \sqrt{54}$

$d(E, BG) = EE' = \sqrt{54} \approx 7{,}35$



e $d(F, MHG) = d(F, NG) = FF'$ met F' op GN zo dat $FF' \perp GN$.



$NG = \sqrt{6^2 + 3^2} = \sqrt{45}$

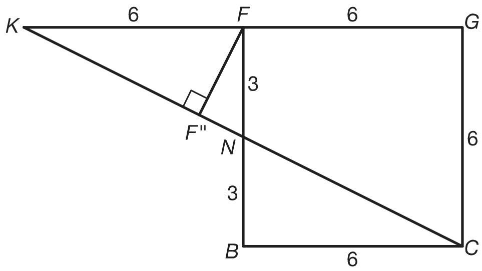
Zijde × hoogte-methoden in $\triangle NGF$ geeft $NG \times FF' = FN \times FG$

$$\sqrt{45} \times FF' = 3 \times 6$$

$$FF' = \frac{18}{\sqrt{45}}$$

Dus $d(F, MHG) = FF' = \dfrac{18}{\sqrt{45}} \approx 2{,}68$.

f $d(F, CDM) = d(F, CN) = FF''$ met F'' op CN zo dat $FF'' \perp CN$.



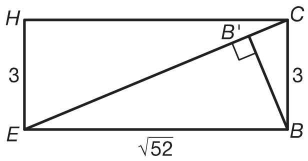
FF'' is net zo lang als FF' bij e.

Dus $d(F, CDM) = FF'' \approx 2{,}68$.

50 **a** $d(M, AB) = d(G, AB) = GB = \sqrt{3^2 + 4^2} = 5$

b $d(B, CE)$ in vlak $EBCH$.

$EB = \sqrt{6^2 + 4^2} = \sqrt{52}$



$EC = \sqrt{52 + 3^2} = \sqrt{61}$

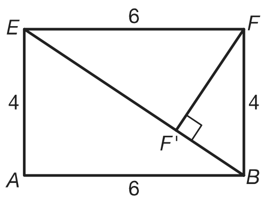
In $\triangle EBC$ zijde × hoogte-methode geeft $EC \times BB' = EB \times BC$

$$\sqrt{61} \times BB' = \sqrt{52} \times 3$$

$$BB' = \frac{3\sqrt{52}}{\sqrt{61}}$$

$d(B, CE) = BB' = \dfrac{3\sqrt{52}}{\sqrt{61}} \approx 2{,}77$

c $d(F, BCH) = d(F, BE) = FF'$ met F' op EB zo dat $FF' \perp EB$



$BE = \sqrt{52}$

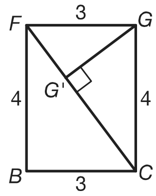
In $\triangle BFE$ zijde × hoogte-methode: $EB \times FF' = EF \times BF$

$$\sqrt{52} \times FF' = 6 \times 4$$

$$FF' = \frac{24}{\sqrt{52}}$$

Dus $d(F, BCH) = FF' = \frac{24}{\sqrt{52}} \approx 3{,}33$.

d $d(M, DEF) = d(M, DEFC) = d(G, DEFC) = d(G, CF) = GG'$ met G' op FC zo dat $GG' \perp FC$.



$FC = \sqrt{9+16} = 5$

In $\triangle CGF$ zijde × hoogte-methode: $FC \times GG' = CG \times FG$

$$5 \times GG' = 4 \times 3$$

$$GG' = \tfrac{12}{5} = 2\tfrac{2}{5}$$

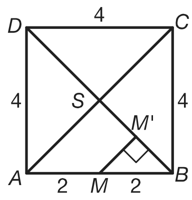
Dus $d(M, DEF) = GG' = 2\tfrac{2}{5}$.

51 **a** $d(M, BDT) = d(M, BD) = MM'$ met M' op BD zo dat $MM' \perp BD$.

$MM' = \tfrac{1}{2} \cdot AS = \tfrac{1}{4} AC$

$AC = \sqrt{4^2 + 4^2} = \sqrt{32}$

Dus $d(M, BDT) = MM' = \tfrac{1}{4}\sqrt{32} \approx 1{,}41$.



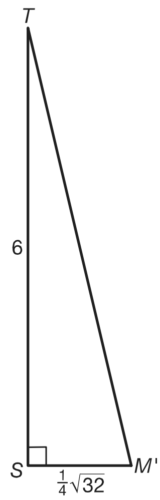
b MN snijdt BD in M' met M' het midden van BS.

$BD = \sqrt{4^2 + 4^2} = \sqrt{32}$

$BS = \tfrac{1}{2}\sqrt{32}$

$SM' = \tfrac{1}{4}\sqrt{32}$

$d(T, MN) = TM' = \sqrt{6^2 + \left(\tfrac{1}{4}\sqrt{32}\right)^2} = \sqrt{36 + \tfrac{1}{16} \cdot 32} = \sqrt{38} \approx 6{,}16$



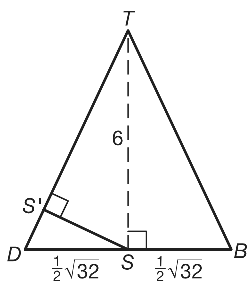
c $DT = \sqrt{\tfrac{1}{4} \cdot 32 + 36} = \sqrt{44}$

In $\triangle DST$ zijde × hoogte-methode: $DT \times SS' = DS \times ST$

$$\sqrt{44} \times SS' = \tfrac{1}{2}\sqrt{32} \times 6$$

$$SS' = \frac{3\sqrt{32}}{\sqrt{44}}$$

Dus $d(S, DT) = \frac{3\sqrt{32}}{\sqrt{44}} \approx 2{,}56$.



d $d(M, CDT)$ ligt in het vlak door M en ST.

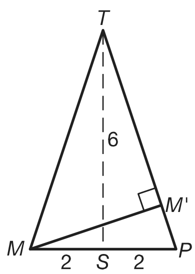
$PT = \sqrt{4 + 36} = \sqrt{40}$

In $\triangle MPT$ zijde × hoogte-methode: $PT \times MM' = MP \times ST$

$$\sqrt{40} \times MM' = 4 \times 6$$

$$MM' = \frac{24}{\sqrt{40}}$$

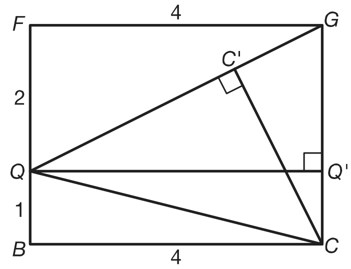
Dus $d(M, CDT) = MM' = \frac{24}{\sqrt{40}} \approx 3{,}79$.



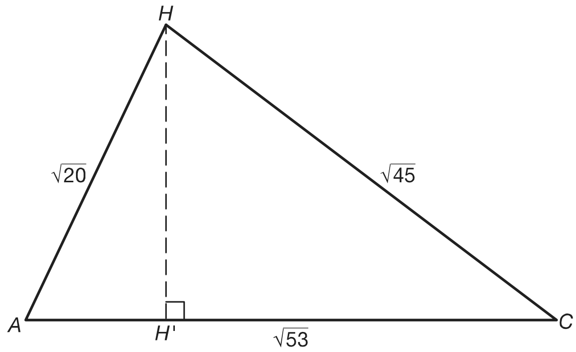
e $d(A, CDT) = d(M, CDT) \approx 3{,}79$

f $d(A, BCT) = d(A, CDT) \approx 3{,}79$

52 **a** $QG = \sqrt{4^2 + 2^2} = \sqrt{20}$
$QG \times CC' = CG \times QQ'$
$\sqrt{20} \times CC' = 3 \times 4$
$CC' = \dfrac{12}{\sqrt{20}} \approx 2{,}68$



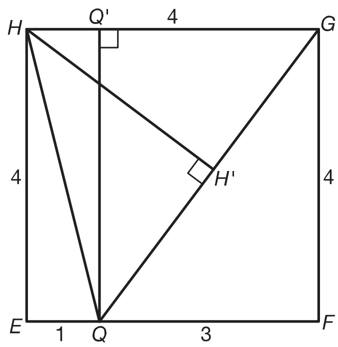
b



$AC = \sqrt{6^2 + 4^2} = \sqrt{52}$
In rechthoek $ACGE$ geeft de stelling van Pythagoras in
$\triangle ACP$: $PC = \sqrt{52+1} = \sqrt{53}$
$HP = \sqrt{4^2 + 2^2} = \sqrt{20}$
$CH = \sqrt{6^2 + 3^2} = \sqrt{45}$
$CH^2 = PC^2 + HP^2 - 2 \cdot PC \cdot HP \cdot \cos \angle CPH$
$45 = 53 + 20 - 2 \cdot \sqrt{53} \cdot \sqrt{20} \cdot \cos \angle CPH$
$2 \cdot \sqrt{53} \cdot \sqrt{20} \cdot \cos \angle CPH = 28$
$\cos \angle CPH = \dfrac{28}{2 \cdot \sqrt{53} \cdot \sqrt{20}}$
$\angle CPH \approx 64{,}5°$

c $\sin \angle CPH = \dfrac{HH'}{HP}$
$HH' = HP \cdot \sin \angle CPH = \sqrt{20} \times \sin 64{,}5°$
$HH' \approx 4{,}04$

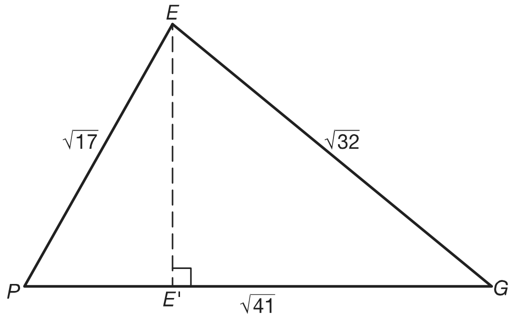
53 **a** $d(H, PCG) = d(H, GQ) = HH'$ met H' op GQ zo dat $HH' \perp GQ$.
$QG = \sqrt{3^2 + 4^2} = 5$
In $\triangle HGQ$ zijde × hoogte-methode:
$QG \times HH' = HG \times QQ'$
$HH' = \dfrac{4 \times 4}{5} = 3\frac{1}{5}$
$d(H, PCG) = HH' = 3\frac{1}{5}$



b $EP = \sqrt{4^2 + 1^2} = \sqrt{17}$

$EG = \sqrt{4^2 + 4^2} = \sqrt{32}$

$PG = \sqrt{\text{PC}^2 + 4^2} = \sqrt{3^2 + 4^2 + 4^2} = \sqrt{41}$



In $\triangle EPG$: $EG^2 = PG^2 + EP^2 - 2 \cdot PG \cdot EP \cdot \cos \angle P$

$32 = 41 + 17 - 2 \cdot \sqrt{41} \cdot \sqrt{17} \cdot \cos \angle P$

$2 \cdot \sqrt{41} \cdot \sqrt{17} \cdot \cos \angle P = 26$

$\cos \angle P = \dfrac{26}{2 \cdot \sqrt{41} \cdot \sqrt{17}}$

$\angle P \approx 60{,}5°$

In $\triangle PE'E$: $\sin \angle P = \dfrac{EE'}{PE}$

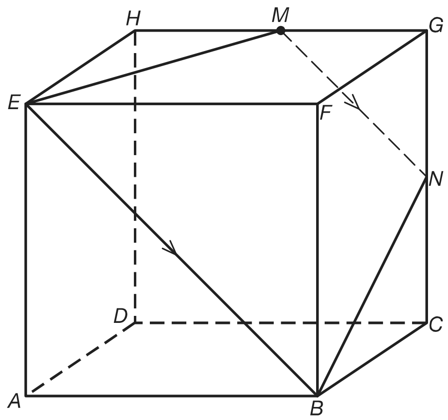
$EE' = PE \cdot \sin \angle P$

$EE' = \sqrt{17} \cdot \sin 60{,}5°$

$EE' \approx 3{,}59$

$d(E, PG) = EE' \approx 3{,}59$

54 **a**

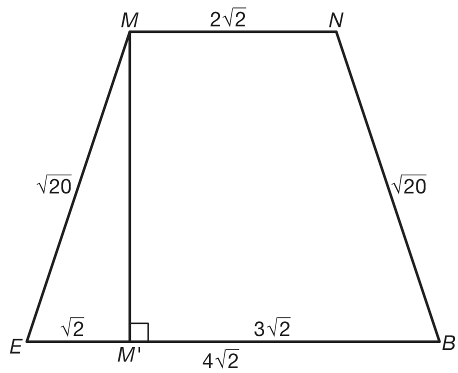


$BE = \sqrt{4^2 + 4^2} = \sqrt{2 \cdot 4^2} = 4\sqrt{2}$

$EM = \sqrt{4^2 + 2^2} = \sqrt{20}$

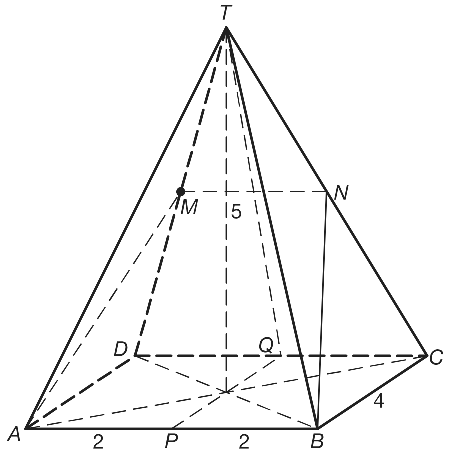
$BN = EM = \sqrt{20}$

$MN = \frac{1}{2} \cdot \text{BE} = \frac{1}{2} \cdot 4\sqrt{2} = 2\sqrt{2}$

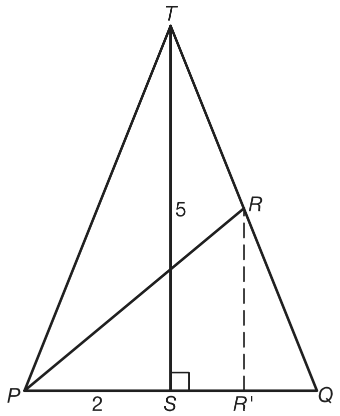


b $d(M, BE) = MM' = \sqrt{20 - 2} = \sqrt{18} \approx 4{,}24$

55 **a**



b



$MN \,//\, AB$ dus $d(M, AB) = d(R, AB) = RP$

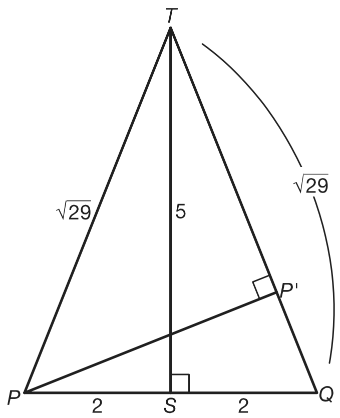
c Trek $RR' \perp PQ$.

R is het midden van QT, dus $RR' = \frac{1}{2} \cdot TS = 2\frac{1}{2}$ en $SR' = \frac{1}{2} \cdot SQ = 1$, dus $PR' = 3$.

$PR = \sqrt{3^2 + \left(2\frac{1}{2}\right)^2} = \sqrt{9 + 6\frac{1}{4}} = \sqrt{15\frac{1}{4}} \approx 3{,}91$

d $d(MN, AB) = d(R, AB) = RP \approx 3{,}91$

e



$QT = \sqrt{2^2 + 5^2} = \sqrt{29}$

In $\triangle PQT$ de zijde × hoogte-methode: $QT \times PP' = PQ \times ST$

$$\sqrt{29} \times PP' = 4 \times 5$$

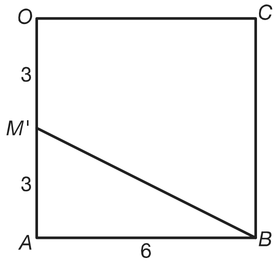
$$PP' = \frac{20}{\sqrt{29}}$$

$d(P, CDT) = PP' = \frac{20}{\sqrt{29}} \approx 3{,}71$

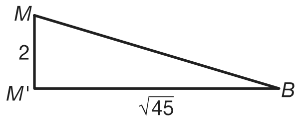
f $AB \,//\, CDT$ dus $d(AB, CDT) = d(A, CDT) = d(B, CDT) = d(P, CDT)$.

56 **a** $QC = \sqrt{2^2 + 6^2} = \sqrt{40} \approx 6{,}32$

b M' is de projectie van M op de Oxy-vlak



$M'B = \sqrt{3^2 + 6^2} = \sqrt{45}$

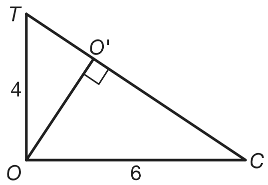


$BM = \sqrt{2^2 + 45} = \sqrt{49} = 7$

c $d(MQ, BC) = d(Q, BC) = QC \approx 6{,}32$ (zie vraag a)

d $d(OA, BCT) = d(O, BCT) = d(O, CT) = OO'$

met O' op CT zo dat $OO' \perp CT$.



$CT = \sqrt{4^2 + 6^2} = \sqrt{52}$

In $\triangle OCT$ de zijde $\times$ hoogte-methode:

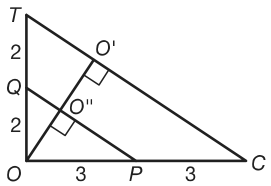
$CT \times OO' = OC \times OT$

$\sqrt{52} \times OO' = 6 \times 4$

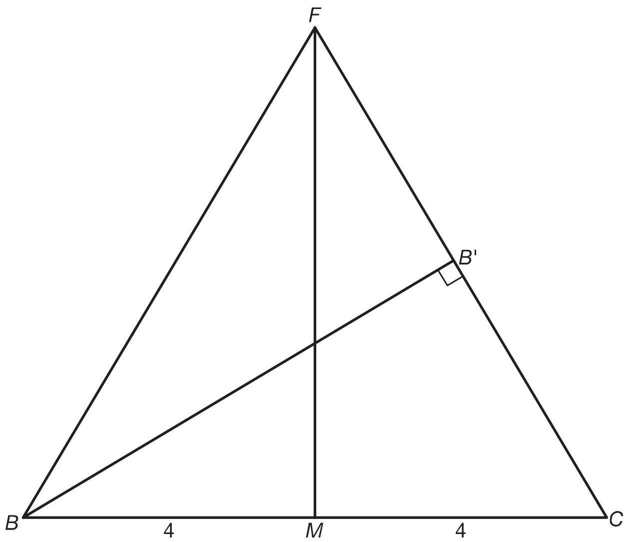
$OO' = \dfrac{24}{\sqrt{52}} \approx 3{,}33$

e $d(MNP, BCT) = d(MNPQ, BCT) = d(PQ, BCT) = d(PQ, CT)$

$= O''O' = \frac{1}{2} \cdot OO' \approx \frac{1}{2} \cdot 3{,}33 \approx 1{,}66.$



57 **a** $d(B, CF)$ kan in $\triangle BCF$ berekend worden.



In $\triangle F'MF$ geeft de stelling van Pythagoras

$MF = \sqrt{3^2 + 6^2} = \sqrt{45}$

en in $\triangle MCF$: $CF = \sqrt{4^2 + 45} = \sqrt{61}$.

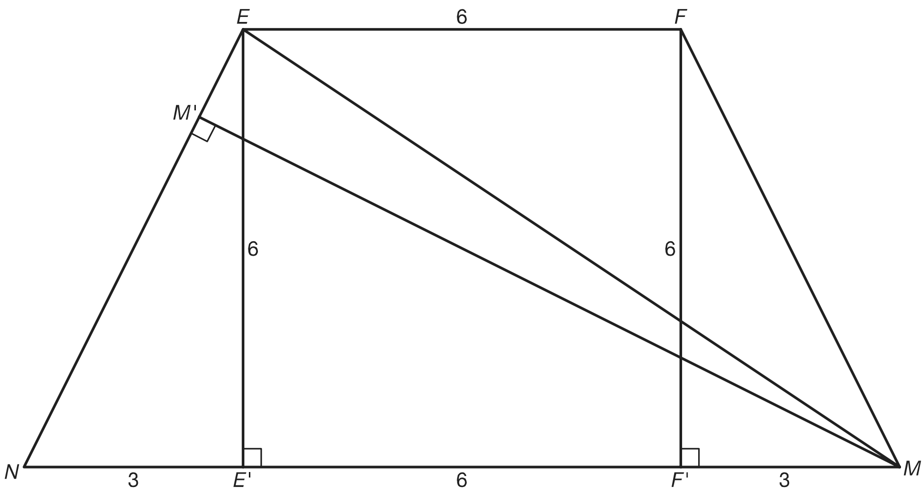
In $\triangle BCF$ de zijde $\times$ hoogte-methode: $CF \times BB' = BC \times MF$

$$\sqrt{61} \times BB' = 8 \times \sqrt{45}$$

$$BB' = \frac{8\sqrt{45}}{\sqrt{61}}$$

$d(B, CF) = BB' = \dfrac{8\sqrt{45}}{\sqrt{61}} \approx 6{,}87$ m

b $d(B, ADE) = d(M, ADE) = MM'$ met M' op EN zo dat $MM' \perp EN$.



$EN = MF = \sqrt{45}$

In $\triangle NME$ geeft de zijde $\times$ hoogte-methode: $EN \times MM' = NM \times EE'$

$$\sqrt{45} \times MM' = 12 \times 6$$

$$MM' = \frac{72}{\sqrt{45}}$$

$d(B, ADE) = MM' = \dfrac{72}{\sqrt{45}} \approx 10{,}73$ m.

c $d(AB, EF) = d(E, AB) = EE''$ met E'' is de loodrechte projectie van E op AB.

In $\triangle E''E'E$: $EE'' = \sqrt{6^2 + 4^2} = \sqrt{52} \approx 7{,}21$

Dus $d(AB, EF) = EE'' \approx 7{,}21$ m.

d $d(E, BCF) = d(E, MF) = ES$ met S is de loodrechte projectie van E op MF.

$TF = MF = \sqrt{45}$

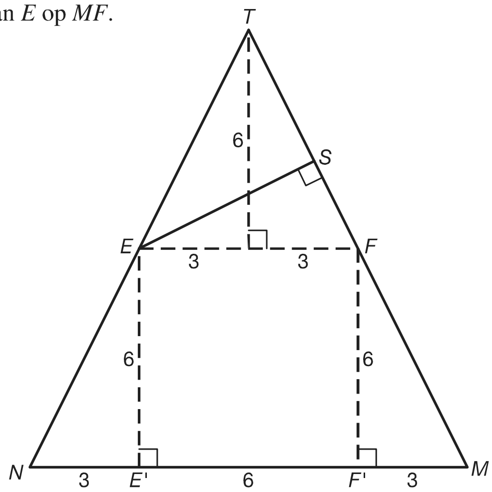
In $\triangle EFT$ de zijde $\times$ hoogte-methode:

$TF \times ES = 6 \times 6$

$\sqrt{45} \times ES = 36$

$ES = \dfrac{36}{\sqrt{45}} \approx 5{,}37$

Dus $d(E, BCF) = ES = 5{,}37$ m.



58 **a** Vlak $DBFH$.

b $NP \mathbin{//} FM$, dus een snavelfiguur.

c $\triangle BNP \backsim \triangle BFM$.

Omdat $BN : BF = 1 : 2$, is ook $NP : FM = 1 : 2$.

Dit geeft $NP = \frac{1}{2}FM$.

Omdat $FM = \frac{1}{2}FM = \frac{1}{2}BD$, geldt $NP = \frac{1}{2}FM = \frac{1}{4}BD$.

d $NP \mathbin{//} BD$, dus een zandloperfiguur.

e $\triangle NPS \backsim \triangle DBS$.

Omdat $NP : DB = 1 : 4$, is ook $NS : DS = 1 : 4$.

Dit geeft $NS = \frac{1}{5}DN$.

f Met de stelling van Pythagoras in $\triangle ABD$: $DB^2 = 3^2 + 3^2 = 18$, dus $DB = \sqrt{18}$.

Met de stelling van Pythagoras in $\triangle DBN$: $DN^2 =$ dus $\left(\sqrt{18}\right)^2 + \left(1\frac{1}{2}\right)^2 = 20\frac{1}{4}$,

dus $DN = \sqrt{20\frac{1}{4}} = 4\frac{1}{2}$.

$NS = \frac{1}{5} \cdot DN = \frac{1}{5} \cdot 4\frac{1}{2} = \frac{9}{10}$

bladzijde 100

59 Stellling van Pythagoras in $\triangle ABD$ geeft

$DB = \sqrt{25+16} = \sqrt{41}$.

Dus $DM = MB = \frac{1}{2}\sqrt{41}$.

In $\triangle DBF$ geeft de stelling van Pythagoras

$DF = \sqrt{41+9} = \sqrt{50}$.

$\triangle DNM \backsim \triangle DBF$ want

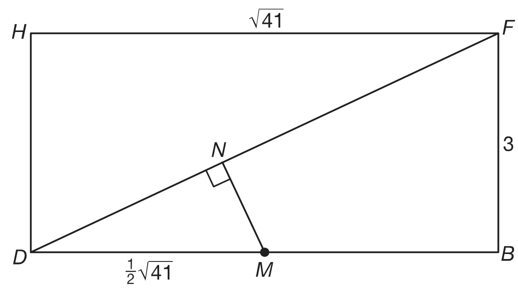
$\angle DNM = \angle DBF = 90°$ en $\angle NDM = \angle BDF$.

MN	DM
BF	DF

geeft

MN	$\frac{1}{2}\sqrt{41}$
3	$\sqrt{50}$

Dus $MN = \dfrac{3 \cdot \frac{1}{2}\sqrt{41}}{\sqrt{50}} \approx 1{,}36$.



bladzijde 101

60 $EQ = QP = PC$, dus $EQ : CQ = 1 : 2$.

$\triangle EQS \backsim \triangle CQG$ (zandloperfiguur).

Omdat $EQ : CQ = 1 : 2$, is ook $ES : CG = 1 : 2$.

Dus is $ES = \frac{1}{2}CG = \frac{1}{2}AE$.

Dus is S het midden van AE.

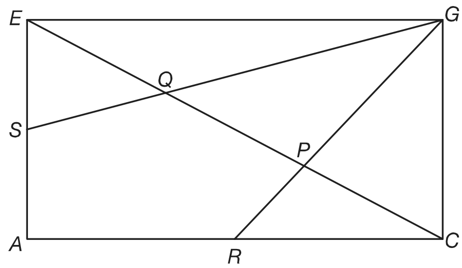
$EQ = QP = PC$, dus is $PC : PE = 1 : 2$

$\triangle PCR \backsim \triangle PEG$ (zandloperfiguur).

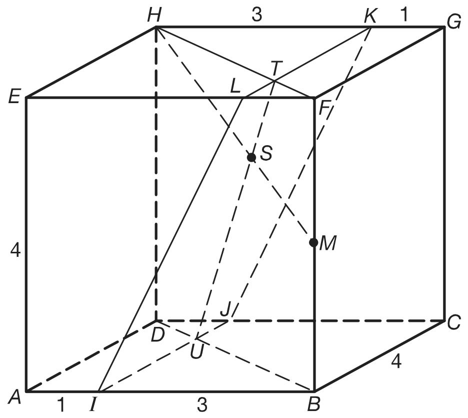
Omdat $PC : PE = 1 : 2$, is ook $CR : EG = 1 : 2$.

Dus is $CR = \frac{1}{2}EG = \frac{1}{2}AC$.

Dus is R het midden van AC.

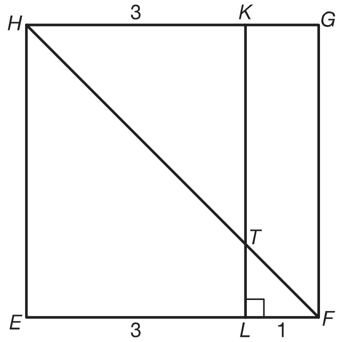


61 **a**

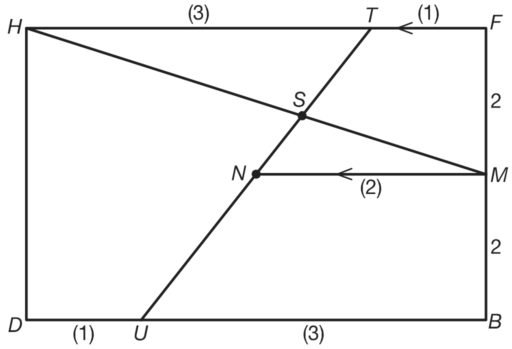


Hulpvlak *DBFH*.
De lijn *UT* is de snijlijn van de vlakken *IJKL* en *DBFH*.
Het punt *S* is het snijpunt van de lijn *HM* met de lijn *UT*.

b

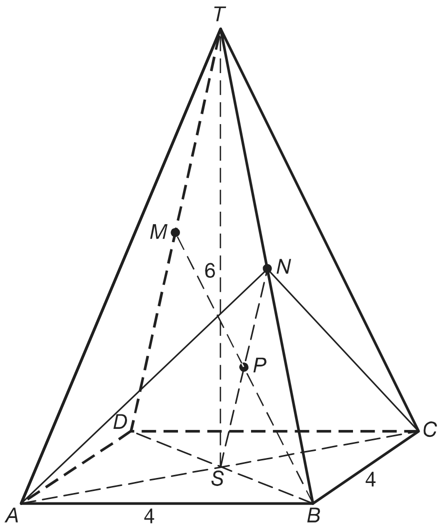


In bovenvlak: $\triangle TLF \backsim \triangle TKH$ (snavelfiguur)
Omdat $LF : KH = 1 : 3$, is ook $TF : TH = 1 : 3$.
Op dezelfde manier in het grondvlak: $UD : UB = 1 : 3$.
De stelling van Pythagoras in $\triangle EFH$: $HF = \sqrt{4^2 + 4^2} = \sqrt{32}$.
De stelling van Pythagoras in $\triangle HFM$: $HM = \sqrt{\left(\sqrt{32}\right)^2 + 2^2} = 6$.

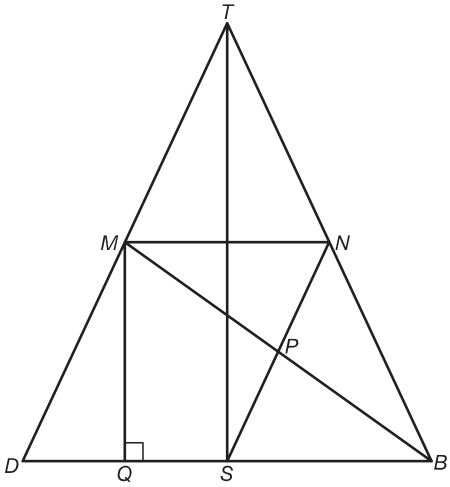


In het hulpvlak *DBFH* is *MN* getekend met *M* op *UT* en *MN* // *HF*.
$\triangle MNS \backsim \triangle HTS$ (zandloperfiguur).
Omdat $MN : HT = 2 : 3$, is ook $MS : HS = 2 : 3$.
Dit geeft $HS = \frac{3}{5} HM$.
Omdat $HM = 6$ volgt hieruit dat $HS = \frac{3}{5} \cdot 6 = \frac{18}{5} = 3\frac{3}{5}$.

62



Hulpvlak *DBT*.
De lijn *SN* is de snijlijn van de vlakken *ACN* en *DBT*.
Het punt *P* is het snijpunt van de lijn *BM* met de lijn *SN*.



De stelling van Pythagoras in $\triangle ABD$: $DB = \sqrt{4^2 + 4^2} = \sqrt{32}$.

$BQ = \frac{3}{4}DB$, dus $BQ = \frac{3}{4}\sqrt{32}$.

$MQ = \frac{1}{2}TS = \frac{1}{2} \cdot 6 = 3$
De stelling van Pythagoras in $\triangle QBM$: $BM = \sqrt{\left(\frac{3}{4}\sqrt{32}\right)^2 + 3^2} = \sqrt{27}$.

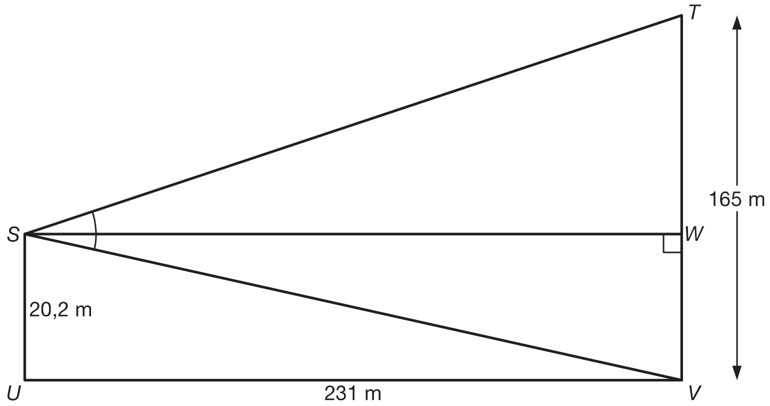
In het hulpvlak *DBT*: $\triangle SBP \backsim \triangle NMP$ (zandloperfiguur).
$SB : NM = 1 : 1$, dus ook $BP : MP = 1 : 1$.
Dit geeft $BP = \frac{1}{2}BM$.

Omdat $BM = \sqrt{27}$, is dus $BP = \frac{1}{2}\sqrt{27} \approx 2{,}60$.

6.6 Diagnostische toets

bladzijde 103

1



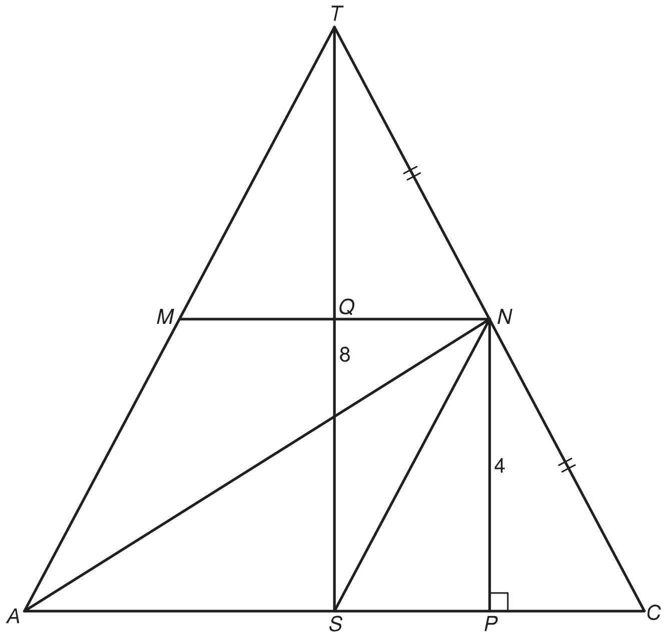
De gevraagde hoek is $\angle VST$.

In $\triangle VSW$: $\tan \angle VSW = \frac{VW}{SW} = \frac{20,2}{231}$ geeft $\angle VSW \approx 5,0°$.

In $\triangle WST$: $\tan \angle WST = \frac{WT}{SW} = \frac{165 - 20,2}{231}$ geeft $\angle WST \approx 32,1°$.

$\angle VST = \angle VSW + \angle WST \approx 5,0° + 32° \approx 37°$

2 **a**



In $\triangle ABC$ geeft de stelling van Pythagoras

$AC^2 = 6^2 + 6^2 = 72$, dus $AC = \sqrt{72}$.

Teken in hulpvlak ACT het lijnstuk NP loodrecht op AC.

N is het midden van CT, dus $NP = \frac{1}{2} \cdot TS = 4$.

$AS = SC = \frac{1}{2}\text{AC} = \frac{1}{2}\sqrt{72}$

en $SP = PC = \frac{1}{2} \cdot \text{SC} = \frac{1}{4}\sqrt{72}$

Dus $AP = AS + SP = \frac{3}{4}\sqrt{72}$.

In $\triangle APN$

$\tan \angle PAN = \frac{\text{NP}}{\text{AP}} = \frac{4}{\frac{3}{4}\sqrt{72}}$

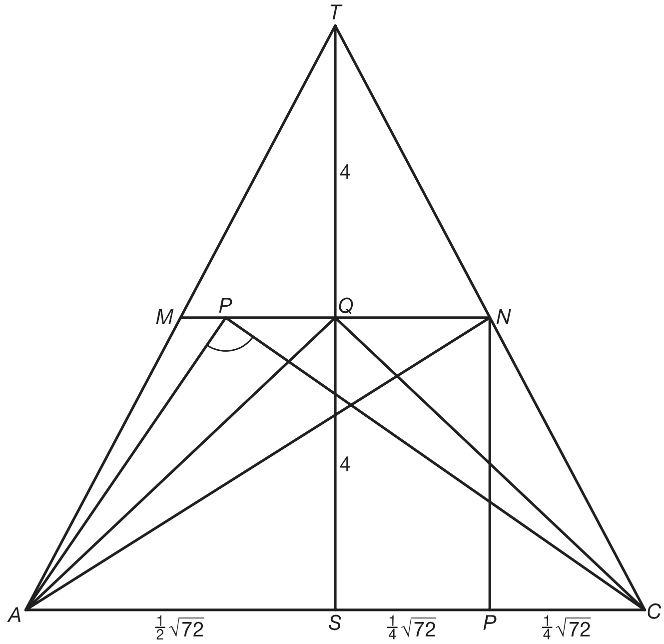
dus $\angle NAC = \angle PAN \approx 32°$

Trek MN en noemt het snijpunt met TS punt Q en trek NS.

Q is het midden van TS, dus $QS = 4$.

$MN = \frac{1}{2} \cdot AC = \frac{1}{2}\sqrt{72}$
$QN = \frac{1}{2} MN = \frac{1}{4}\sqrt{72}$
In $\triangle SNQ$: $\tan \angle MNS = \frac{QS}{QN} = \frac{4}{\frac{1}{4}\sqrt{72}}$
$\angle MNS \approx 62°$

b



$\angle APC$ is maximaal als $P = Q$.

In $\triangle AQS$: $\tan \angle AQS = \frac{AS}{QS} = \frac{\frac{1}{2}\sqrt{72}}{4}$ geeft $\angle AQS \approx 46{,}7°$.
Dus $\angle AQC = 2 \cdot \angle AQS \approx 93°$.
$\angle APC$ is minimaal als $P = N$ of als $P = M$.

In $\triangle ANP$: $\tan \angle ANP = \frac{AP}{NP} = \frac{\frac{3}{4}\sqrt{72}}{4}$ geeft $\angle ANP \approx 57{,}8°$.

In $\triangle PNC$: $\tan \angle PNC = \frac{PC}{NP} = \frac{\frac{1}{4}\sqrt{72}}{4}$ geeft $\angle PNC \approx 27{,}9°$.
Dus $\angle ANC = \angle ANP + \angle PNC \approx 86°$.
Dus $\angle APC$ kan waarden aannemen tussen 86° en 93°.

3 Cosinusregel
$a^2 = b^2 + c^2 - 2bc \cos \alpha$
$3^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos \alpha$
$9 = 36 + 64 - 96 \cos \alpha$
$96 \cos \alpha = 91$
$\cos \alpha = \frac{91}{96}$
$\alpha \approx 18{,}6° \approx 19°$

$b^2 = a^2 + c^2 - 2ac \cos \beta$
$6^2 = 3^2 + 8^2 - 2 \cdot 3 \cdot 8 \cdot \cos \beta$
$36 = 9 + 64 - 48 \cdot \cos \beta$
$48 \cos \beta = 37$
$\cos \beta = \frac{37}{48}$
$\beta \approx 39{,}6° \approx 40°$

$\gamma = 180° - \alpha - \beta = 180° - 18{,}6° - 39{,}6° \approx 121{,}9° \approx 122°$

4 Cosinusregel
$b^2 = a^2 + c^2 - 2ac \cos \beta$
$b^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos 55°$
$b^2 = 100 - 96 \cdot \cos 55°$
$b^2 \approx 44{,}94$
$b \approx \sqrt{44{,}94} \approx 6{,}7$

5 **a** In zijvlak $ABFE$: $BE^2 = AB^2 + AE^2$

$BE^2 = 49 + 16 = 65$

$BE = \sqrt{65}$

In zijvlak $BCGF$: $BG^2 = BC^2 + CG^2 = 36 + 16 = 52$

$BG = \sqrt{52}$

In voorvlak: $EG^2 = EH^2 + HG^2 = 36 + 49 = 85$

$EG = \sqrt{85}$

b In $\triangle BEG$ de cosinusregel $EG^2 = BE^2 + BG^2 - 2 \cdot BE \cdot BG \cos \angle EBG$

$85 = 65 + 52 - 2 \cdot \sqrt{65} \cdot \sqrt{52} \cdot \cos \angle EBG$

$2 \cdot \sqrt{65} \cdot \sqrt{52} \cdot \cos \angle EBG = 32$

$\cos \angle EBG = \dfrac{32}{2\sqrt{65} \cdot \sqrt{52}}$

$\angle EGB \approx 74{,}0°$

c $O(\triangle EBG) = \frac{1}{2} \cdot BE \cdot BG \cdot \sin \angle EBG \approx \frac{1}{2} \cdot \sqrt{65} \cdot \sqrt{52} \cdot \sin 74{,}0° \approx 27{,}9$

bladzijde 104

6 **a** $\angle(AP, CP) = \angle APC$ in $\triangle APC$.

$AP = \sqrt{5^2 + 3^2} = \sqrt{34}$

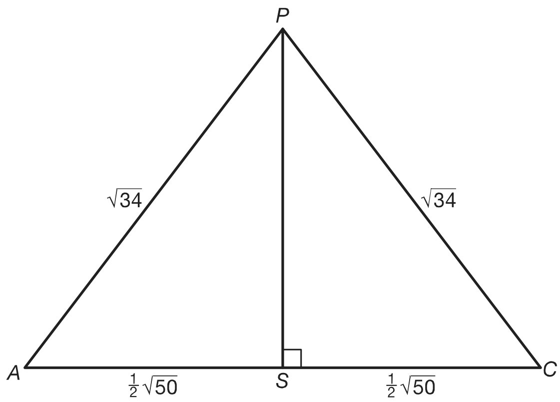
$PC = \sqrt{5^2 + 3^2} = \sqrt{34}$

$AC = \sqrt{5^2 + 5^2} = \sqrt{50}$

$\sin \angle APS = \dfrac{\frac{1}{2}\sqrt{50}}{\sqrt{34}}$

$\angle APS \approx 37{,}3°$

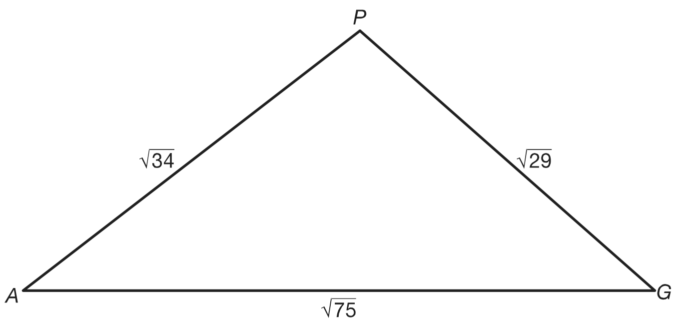
$\angle(AP, CP) = \angle APC = 2 \cdot \angle APS \approx 75°$



b $\angle(APG)$ in $\triangle APG$

$PG = \sqrt{5^2 + 2^2} = \sqrt{29}$

$AG = \sqrt{\mathrm{AC}^2 + \mathrm{CG}^2} = \sqrt{50 + 25} = \sqrt{75}$



$AG^2 = AP^2 + GP^2 - 2 \cdot AP \cdot GP \cdot \cos \angle P$

$75 = 34 + 29 - 2 \cdot \sqrt{34} \cdot \sqrt{29} \cdot \cos \angle P$

$2 \cdot \sqrt{34} \cdot \sqrt{29} \cdot \cos \angle P = -12$

$\cos \angle P = \dfrac{-12}{2 \cdot \sqrt{34} \cdot \sqrt{29}}$

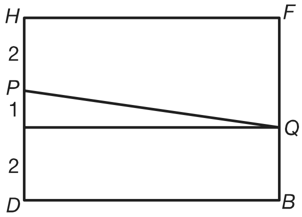
$\angle P \approx 101°$

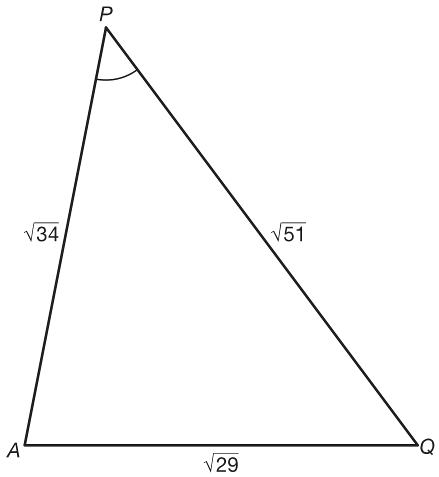
Dus $\angle(AP, PG) = 180° - \angle P \approx 79°$.

c $\angle(AP, PQ) = \angle APQ$ in $\triangle APQ$

$AP = \sqrt{34};\ AQ = \sqrt{5^2 + 2^2} = \sqrt{29}$

$DB = \sqrt{50};\ PQ = \sqrt{50 + 1^2} = \sqrt{51}$





Cosinusregel

$AQ^2 = AP^2 + QP^2 - 2 \cdot AP \cdot QP \cdot \cos \angle APQ$

$29 = 34 + 51 - 2 \cdot \sqrt{34} \cdot \sqrt{51} \cdot \cos \angle APQ$

$2 \cdot \sqrt{34} \cdot \sqrt{51} \cdot \cos \angle APQ = 56$

$\cos \angle APQ = \dfrac{56}{2\sqrt{34} \cdot \sqrt{51}} \approx 0{,}672$

$\angle APQ \approx 48°$.

Dus $\angle(AP, PQ) \approx 48°$.

d $AQ = \sqrt{29};\ QG = \sqrt{34};\ AG = \sqrt{75}$

Cosinusregel

$AG^2 = AQ^2 + QG^2 - 2 \cdot AQ \cdot QG \cdot \cos \angle GQA$

$75 = 29 + 34 - 2 \cdot \sqrt{29} \cdot \sqrt{34} \cos \angle GQA$

$2 \cdot \sqrt{29} \cdot \sqrt{34} \cdot \cos \angle GQA = -12$

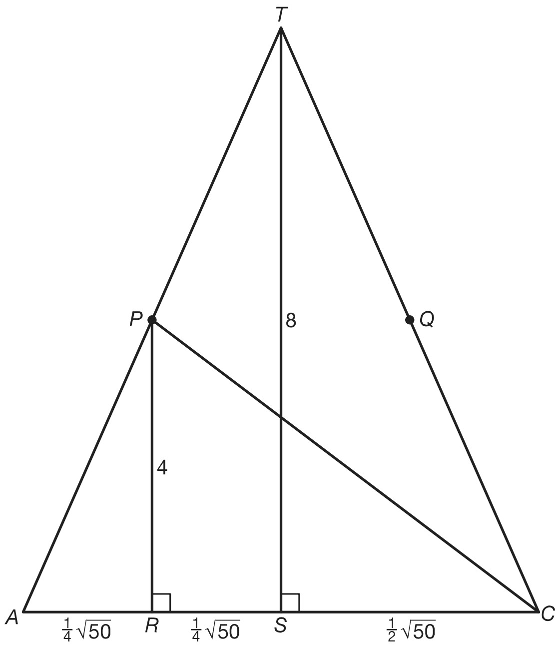
$\cos \angle GQA = -\dfrac{12}{2\sqrt{29} \cdot \sqrt{34}} \approx -0{,}191$

$\angle GQA = 101°$

7 **a** $\angle(PQ, DBT) = 90°$ want $PQ \,//\, AC$ en $AC \perp DBT$.

b $\angle(CP, DBT) = \angle(CP, TS) = \angle(CP, PR) = \angle CPR$

$AC = \sqrt{5^2 + 5^2} = \sqrt{50}$, dus $SC = \frac{1}{2}\sqrt{50}$ en $RS = \frac{1}{4}\sqrt{50}$.



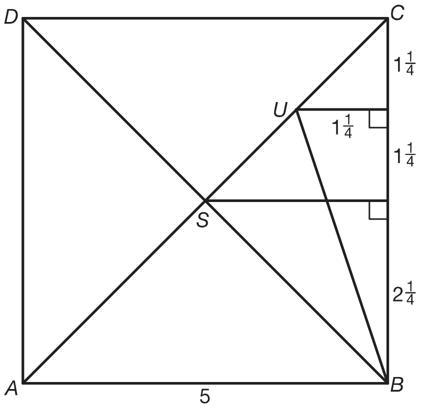
$\tan \angle CPR = \dfrac{\frac{3}{4}\sqrt{50}}{4}$

$\angle CPR \approx 53{,}0°$

Dus $\angle(CP, DBT) \approx 53°$.

c De hellingshoek van AQ = de hellingshoek van $CP = \angle PCA$.
Zie de figuur bij b.
$\angle PCA = 90° - \angle CPR = 90° - 53° = 37°$
Dus de hellingshoek van AQ is 37°.

d Noem het midden van SC punt U.
De hellingshoek van $BQ = \angle(BQ, BU) = \angle QBU$.



$BU = \sqrt{\left(3\frac{3}{4}\right)^2 + \left(1\frac{1}{4}\right)^2} \approx 3{,}95$

$\tan \angle QBU = \frac{QU}{BU} = \frac{4}{3{,}95}$

$\angle QBU \approx 45{,}3°$

Dus de hellingshoek van BQ is 45°.

8 **a** $\angle(ABK, ABC)$ in standvlak $BCKJ$.

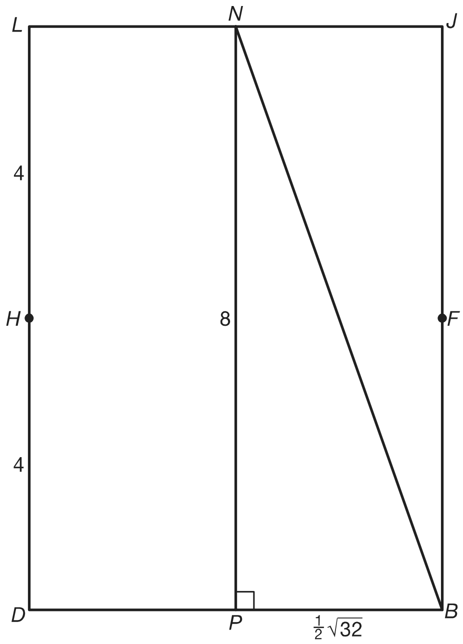
$\tan \angle KBC = \frac{8}{4} = 2$

$\angle KBC \approx 63{,}4°$

Dus $\angle(ABK, ABC) \approx 63°$.

b De hellingshoek van BIK is $\angle NBD$ waarbij N het snijpunt is van IK en LJ.

$DB = \sqrt{4^2 + 4^2} = \sqrt{32}$

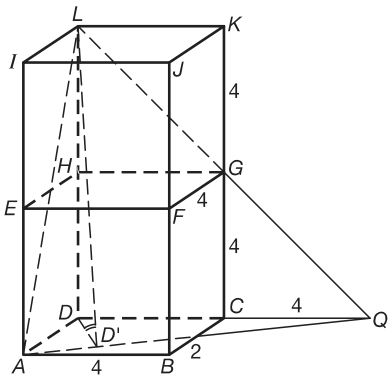


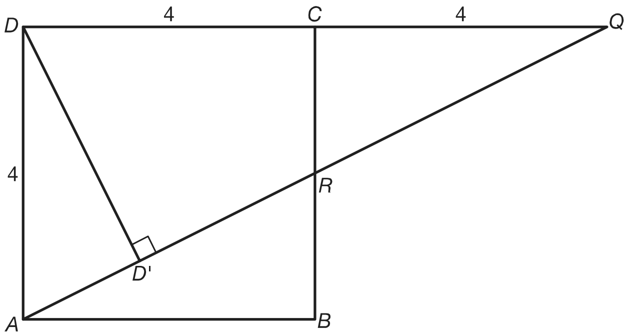
$\tan \angle NBD = \frac{8}{\frac{1}{2}\sqrt{32}}$

$\angle NBD \approx 70{,}53°$

De hellingshoek van BIK is dus ongeveer 71°.

c





Trek DD' loodrecht op AQ.

$AQ = \sqrt{4^2 + 8^2} = \sqrt{80}$

In $\triangle ADQ$ de zijde × hoogte-methode: $AQ \times DD' = AD \times DQ$

$\sqrt{80} \times DD' = 4 \times 8$

$DD' = \dfrac{32}{\sqrt{80}} \approx 3{,}58$

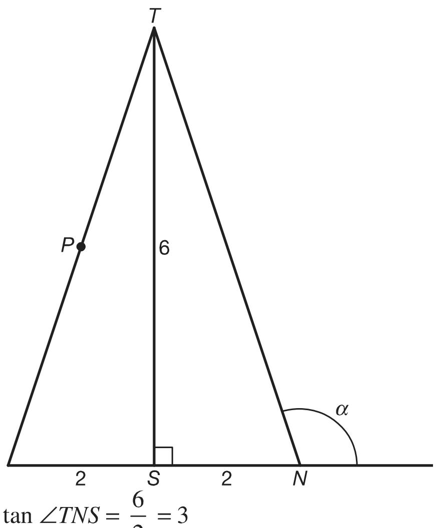
$\tan \angle LD'D = \dfrac{8}{3{,}58}$

$\angle LD'D \approx 65{,}9°$

Dus de hellingshoek van AGL is ongeveer 66°.

bladzijde 105

9 **a** Verticaal hulpvlak door T en het midden N van BC.



$\tan \angle TNS = \dfrac{6}{2} = 3$

$\angle TNS \approx 71{,}6°$

$\alpha = 180° - \angle TNS \approx 108{,}4° \approx 108°$

b Lengte $= \dfrac{108{,}4}{360} \cdot 2\pi \cdot NT = \dfrac{108{,}4}{360} \cdot 2\pi \cdot \sqrt{2^2 + 6^2} \approx 11{,}97$

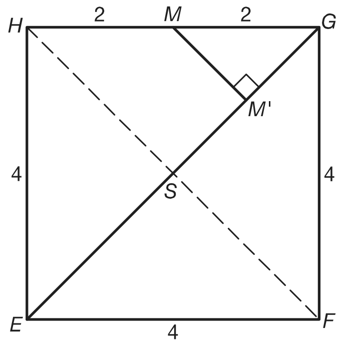
De lengte van de rotatiebaan is dus ongeveer 12,0 dm.

c De lengte van de rotatiebaan van M is gelijk aan de lengte van de baan van het punt P 1

$PN = \sqrt{3^2 + 3^2} = \sqrt{18}$

Dus de lengte van de baan is $\frac{108,4}{360} \cdot 2\pi \cdot \sqrt{18} \approx 8,0$ dm.

10 **a** $d(M, EG) = MM'$ met M' op EG zo dat $MM' \perp EG$.



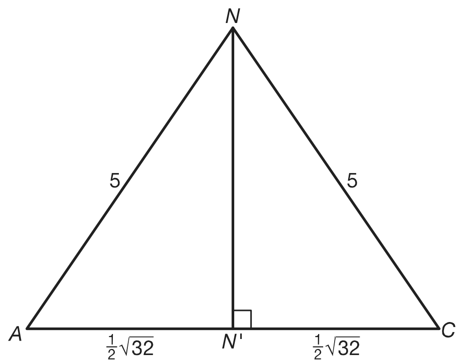
$FH = \sqrt{4^2 + 4^2} = \sqrt{32}$

$SH = \frac{1}{2} \cdot \text{FH} = \frac{1}{2}\sqrt{32}$

$MM' = \frac{1}{2} \cdot \text{SH} = \frac{1}{2} \cdot \frac{1}{2}\sqrt{32} = \frac{1}{4}\sqrt{32}$

$d(M, EG) = MM' = \frac{1}{4}\sqrt{32} \approx 1,41$

b



$d(N, AC) = NN'$ met N' op AC zo dat $NN' \perp AC$.

$NA = \sqrt{4^2 + 3^2} = 5$

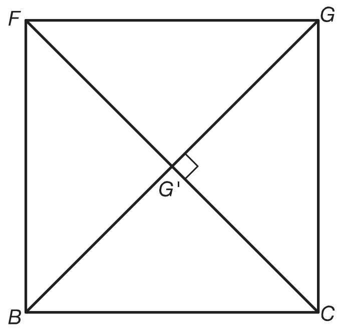
$NC = \sqrt{4^2 + 3^2} = 5$

$AC = \sqrt{4^2 + 4^2} = \sqrt{32}$

$NN' = \sqrt{5^2 - \left(\frac{1}{2}\sqrt{32}\right)^2} = \sqrt{25 - \frac{1}{4} \cdot 32} = \sqrt{17}$

$d(N, AC) = NN' = \sqrt{17} \approx 4,12$

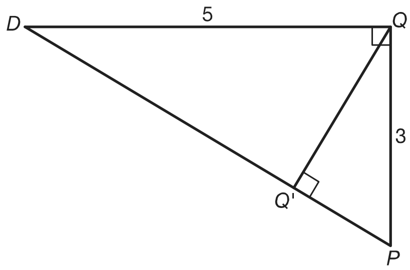
c $d(M, EDC) = d(GH, EDCF) = d(G, EDCF) = d(G, CF)$



$d(G, CF) = GG' = \frac{1}{2} \cdot \text{GB} = \frac{1}{2}\sqrt{32}$

Dus $d(M, EDC) = \frac{1}{2}\sqrt{32} \approx 2,83$.

11 **a** $d(PG, ABC) = d(G, ABC) = GC = 4$



b $d(Q, DP)$ in $\triangle DPQ$.

$PQ = 3$

$DQ = \sqrt{4^2 + 3^2} = 5$

$DP = \sqrt{5^2 + 3^2} = \sqrt{34}$

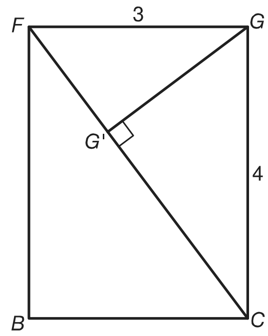
De zijde × hoogte-methode geeft

$DP \times QQ' = DQ \times PQ$

$\sqrt{34} \times QQ' = 5 \times 3$

$QQ' = \frac{15}{\sqrt{34}}$

Dus $d(Q, DP) = \frac{15}{\sqrt{34}} \approx 2{,}57$.



c $d(GH, EDC) = d(GH, EDCF) = d(G, EDCF) = d(G, CF)$

$FC = \sqrt{3^2 + 4^2} = 5$

De zijde × hoogte-methode in $\triangle CGF$ geeft

$FC \times GG' = CG \times FG$

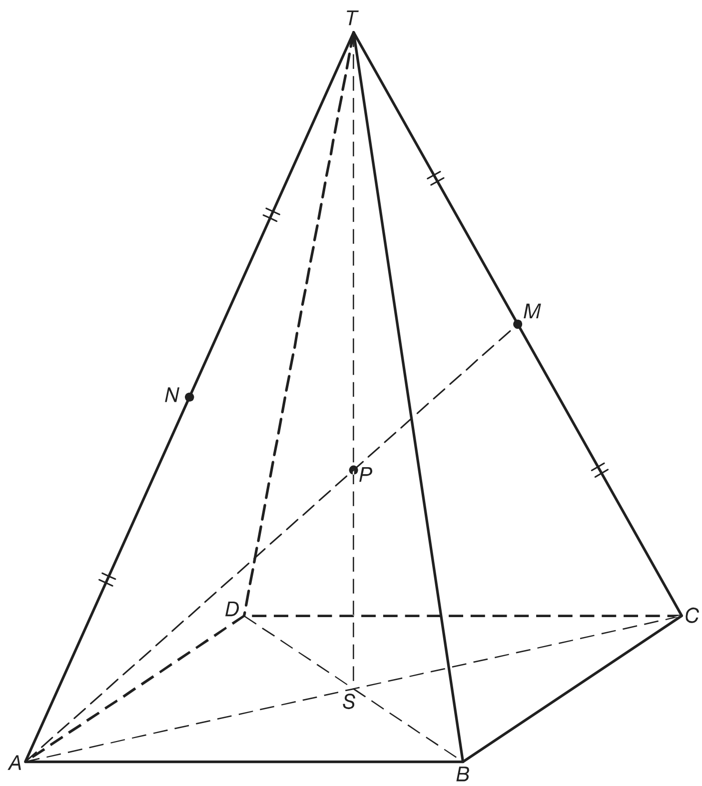
$5 \times GG' = 4 \times 3$

$GG' = \frac{12}{5}$

Dus $d(GH, EDC) = \frac{12}{5}$.

d $d(BC, PQ) = d(P, BC) = BP = \sqrt{4^2 + 2^2} = \sqrt{20} \approx 4{,}47$

12 **a**



Lijn AM ligt in het hulpvlak ACT.

De lijn TS is de snijlijn van de vlakken BDT en ACT.

Het punt P is het snijpunt van de lijn AM met de lijn TS.

In hulpvlak ACT: teken hulplijn MQ met Q op TS en $MQ \parallel AC$.

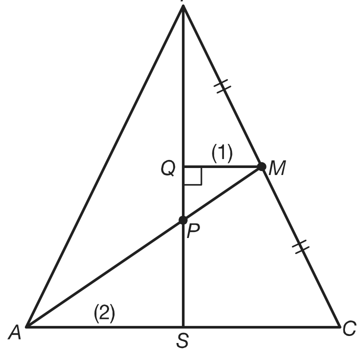
$\triangle APS \backsim \triangle MPQ$ (zandloperfiguur).

Omdat $AS : MQ = 2 : 1$, is ook $PS : PQ = 2 : 1$.

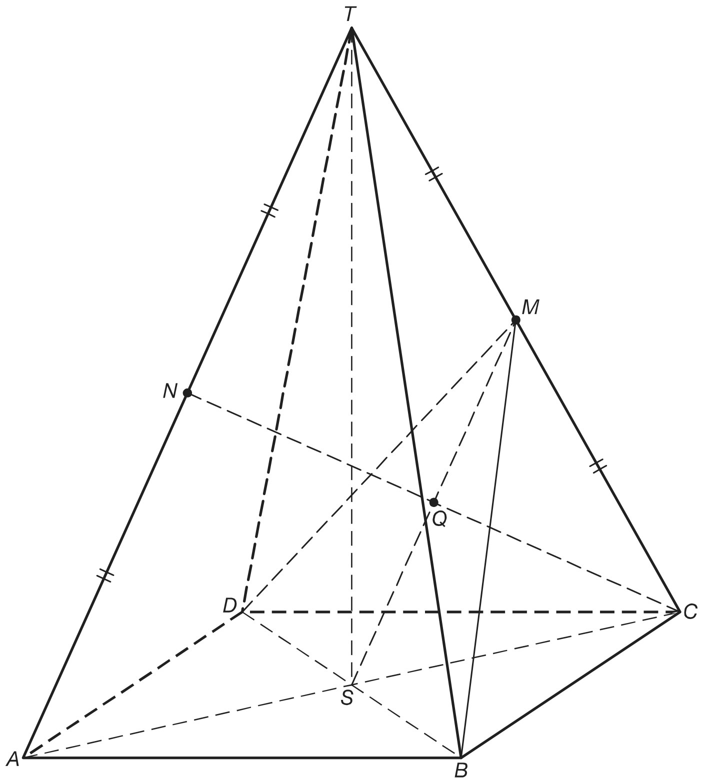
Dit geeft $PS = \frac{2}{3}QS$.

Omdat $QS = \frac{1}{2}TS$ geldt dus

$PS = \frac{2}{3} \cdot \frac{1}{2}TS = \frac{1}{3}TS = \frac{1}{3} \cdot 9 = 3$.



b



CN ligt in het hulpvlak ACT.
De lijn MS is de snijlijn van de vlakken BDM en ACT.
Het punt Q is het snijpunt van de lijn CN met de lijn MS.
In hulpvlak ACT: $\triangle SCQ \backsim \triangle MNQ$.
Omdat $SC : MN = 1 : 1$, is ook $CQ : NQ = 1 : 1$.
Dit geeft $CQ = \frac{1}{2}CN$.
N' is het midden van AS.
$N'C = \frac{3}{4}\text{AC} = \frac{3}{4}\sqrt{6^2 + 6^2} = \frac{3}{4}\sqrt{72}$.

De stelling van Pythagoras in $\triangle NN'C$: $CN = \sqrt{\left(\frac{3}{4}\sqrt{72}\right)^2 + \left(4\frac{1}{2}\right)^2} = \sqrt{60\frac{3}{4}}$.

Dus $CQ = \frac{1}{2}\text{CN} = \frac{1}{2}\sqrt{60\frac{3}{4}} \approx 3{,}90$.